Flying on Sunshine Questions

1 The Sun emits light with a peak wavelength of around 500nm.

a) Calculate the frequency.

$f = c/\lambda = 3.0 \times 10^8 / 500 \times 10^{-9} = \underline{\underline{6.0 \times 10^{14}\ \text{Hz}}}$

b) Calculate the photon energy in Joules.

$E = hf = 6.6 \times 10^{-34} \times 6.0 \times 10^{14} = \underline{\underline{3.96 \times 10^{-19}\ \text{J}}}$

c) Calculate the photon energy in eV.

There are 1.6×10^{-19} J/eV $\therefore$ $E = 3.96 \times 10^{-19}\ \text{J} / 1.6 \times 10^{-19}\ \text{J/eV} = \underline{\underline{2.48\ \text{eV}}}$

2 A photovoltaic cell can produce an e.m.f. of 0.5V.

a) What energy is supplied to each electron by each photon in eV?

The definition of the eV is the energy transferred per electron per volt so = $\underline{\underline{0.5\text{eV}}}$

b) Calculate the energy supplied to each electron in Joules.

$E = QV = 1.6 \times 10^{-19} \times 0.5 = \underline{\underline{8.0 \times 10^{-20}\ \text{J}}}$

c) Calculate the maximum wavelength of light that the photovoltaic cell can make use of.

$E = hc/\lambda \therefore \lambda = hc/E = 6.6 \times 10^{-34} \times 3.0 \times 10^8 / 8.0 \times 10^{-20} = 2.48 \times 10^{-6}\ \text{m} = \underline{\underline{2480\ \text{nm}}}$

d) Explain why light with a longer wavelength will produce no e.m.f from the cell.

Each individual photon has less energy than is required to promote an electron to a higher energy level.

3 Carbon fibre, used in Solar Impulse's wings is low density, stiff and strong.
Explain the meaning of the terms **and** suggest why each property is desirable.
Include the terms Young modulus and yield stress in your explanations.

a) low density density = mass / volume. It is a measure of the amount of matter per unit volume. Low density is desirable because the mass of the plane will be lower. It will therefore require less lift for steady flight reducing the power and hence energy needed for flight.

b) stiff Stiffness is the opposite of flexibility. A stiff material means less can be used to make the winds stiff enough to withstand the forces involved in flight. Less material means the lighter wings. Stiffness can be quantified in terms of the Young modulus which is stress / strain.

c) strong A strong material has a high yield stress. It can withstand a high force/ cross sectional area ratio before yield, and hence, plastic deformation occurs. Wings need to be strong as they must support the weight of the plane and the drag produced by air resistance.

4 Use the **Solar Impulse 2 Data** and data in the text to calculate:

a) The length of the sides of the solar cells (assuming they are square).

area = length of sides^2 $\therefore$ length of side = $\sqrt{\text{area}} = \sqrt{270\text{m}^2} = \underline{\underline{16.4\ \text{m}}}$

b) The energy that can be stored in the batteries.

energy = energy density x mass = $9.4 \times 10^5\ \text{Jkg}^{-1} \times 630\ \text{kg} = \underline{\underline{5.9 \times 10^8\ \text{J}}}$

c) The mean width of the wings.

area = length × width $\therefore$ width = area / length = $270\ \text{m}^2 / 72\text{m} = \underline{\underline{3.75\ \text{m}}}$

d) The gravitational potential energy lost during the 4 hour glide.

$\Delta E_{grav} = mg\Delta h = 2300\ \text{kg} \times 9.81 \times (8500\ \text{m} - 1500\ \text{m}) = \underline{\underline{1.58 \times 10^8\ \text{J}}}$

e) The power at which GPE is lost during the glide.

$P = E/t = 1.58 \times 10^8\ \text{J} / (4 \times 60 \times 60) = 1.10 \times 10^4\ \text{W} = \underline{\underline{11\ \text{kW}}}$

f) Make a sensible estimate as to the power required to keep the plane aloft.

It must be around the same power as the glide down. Say 10 kW.

g) Calculate the solar energy input required to supply this power.

electricity out = 0.23 x solar input $\therefore$ solar input = electricity out / 0.23 = $1 \times 10^4 / 0.23 = \underline{\underline{4.3 \times 10^4\ \text{W}}}$

h) Calculate an estimate of the solar radiation flux in Wm^{-2}.

flux = power / area = $4.3 \times 10^4\ \text{W} / 270\ \text{m}^2 = \underline{\underline{160\ \text{Wm}^{-2}}}$

i) Calculate the time required to fully charge the storage batteries.

$P = E/t \therefore t = E/P = 5.9 \times 10^8\ \text{J} / 11\ \text{kW} = 5.36 \times 10^4\ \text{s} = \underline{\underline{14.9\ \text{hours}}}$

5 Juno uses a gravitational slingshot to increase its velocity.

a) Calculate the kinetic energy gained by Juno.

$\Delta E_k = 0.5 \times m\Delta v^2 = 0.5 \times 3600 \times (4.2 \times 10^4 - 3.5 \times 10^4)^2 = \underline{\underline{8.82 \times 10^{10}\ J}}$

b) Explain the source of this energy.

The Earth (It will have lost this amount of kinetic energy)

c) Calculate the change in momentum of Juno.

$\Delta p = m\Delta v = 3600 \times (4.2 \times 10^4 - 3.5 \times 10^4) = \underline{\underline{2.52 \times 10^7\ kgms^{-1}}}$

d) Explain how the total momentum is conserved.

The Earth will have lost the same amount of momentum as its orbital velocity is reduced.

e) The slingshot took around 9 hours. Calculate the mean gravitational force on Juno.

$F = \Delta mv/\Delta t = 3600 \times (4.2 \times 10^4 - 3.5 \times 10^4) / (9 \times 60 \times 60) = \underline{\underline{780\ N}}$

6 Light intensity follows in inverse square law.

a) Calculate what fraction of the Earth's solar flux that Juno receives.

$=1/5.2^2 = 1/27 = \underline{0.0370}$

b) Calculate power supplied by the solar cells assuming an efficiency of 35%

At Earth solar flux $\approx 160\ Wm^{-2}$ (from 4h) $\therefore$ at Jupiter $= 160 \times 0.0370 = \underline{\underline{5.92\ Wm^{-2}}}$

Area $= 60\ m^2$ $\therefore$ power $= 60\ m \times 5.92\ Wm^{-2} = \underline{\underline{355\ W}}$

7 Juno will orbit Jupiter, which has a mass of 1.9×10^{27} kg, once every 14 Earth days.

a) Calculate the orbital period in seconds.

$T = 14 \times 24 \times 60 \times 60 = 1.21 \times 10^6\ s$

b) Use the equations for centripetal acceleration and gravitational force and the data above to **show that** the average radius of Juno's orbit around Jupiter is about 1.7 million kilometres.

$F = mv^2/r$ and $F = GMm/r^2 \therefore mv^2/r = GMm/r^2 \therefore v^2 = GM/r$ & $v = s/t = 2\pi r/T \therefore v^2 = 4\pi^2 r^2/T^2$

Equating the two expressions for v^2 gives: $GM/r = 4\pi^2 r^2/T^2$ which rearranges to $r^3 = GMT^2/4\pi^2$

$r^3 = 6.67 \times 10^{-11} \times 1.9 \times 10^{27} \times (1.21 \times 10^6)^2 = 4.79 \times 10^{27}\ m^3 \therefore r = \sqrt[3]{4.79 \times 10^{27}} = \underline{\underline{1.68 \times 10^9\ m}}$

c) Calculate the velocity of Juno in its orbit around Jupiter.

$v = 2\pi r/T = 2\pi \times 1.68 \times 10^9\ m / 1.21 \times 10^6\ s = 8.72 \times 10^3\ ms^{-1}$

d) Jupiter has a diameter of 143000 km. Calculate the speed that Juno will enter the top of Jupiter's atmosphere.

ΔE_{grav} will be converted to ΔE_k as Juno falls giving it additional velocity.

In orbit $E_{grav} = -GMm/r_{orbit} = -6.67 \times 10^{-11} \times 1.9 \times 10^{27} \times 3600 / 1.68 \times 10^9 = -2.72 \times 10^{11}\ J$

At top of atmos $E_{grav} = -GMm/r_{atmos} = -6.67 \times 10^{-11} \times 1.9 \times 10^{27} \times 3600 / 1.43 \times 10^8 = -3.19 \times 10^{12}\ J$

$\Delta E_k = -\Delta E_{grav} = -(-3.19 \times 10^{12} - -2.72 \times 10^{11}) = 2.92 \times 10^{12}\ J$

$\Delta E_k = m\Delta v^2/2 \therefore \Delta v = \sqrt{(2\Delta E_k/m)} = \sqrt{(2 \times 2.92 \times 10^{12} / 3600)} = 4.03 \times 10^4\ ms^{-1}$

Final velocity = initial + $\Delta v = 8.72 \times 10^3 + 4.03 \times 10^4 = \underline{\underline{4.9 \times 10^4\ ms^{-1}}}$