① Q1 Hunky Dory (1971)

Q2 a) $g = \frac{GM}{r^2}$ $\therefore$ $r = \sqrt{\frac{GM}{g}} = \sqrt{\frac{6\cdot67\times10^{-11}\times6\cdot4\times10^{23}}{3\cdot7}}$

$$\underline{\underline{r = 3\cdot40\times10^{6}\ \text{m} = 3400\ \text{km}}}$$

b) $V_{grav} = \frac{-GM}{r} = \frac{-6\cdot67\times10^{-11}\times6\cdot4\times10^{23}}{3390\times10^{3}}$

$$= \underline{\underline{-1\cdot26\times10^{7}\ \text{J kg}^{-1}}}$$

c) $E_{grav} = \frac{-GMm}{r}$

at surface $E_{grav} = \frac{-6\cdot67\times10^{-11}\times6\cdot4\times10^{23}\times2500}{3390\times10^{3}}$

$$= -3\cdot15\times10^{10}\ \text{J}$$

in orbit $E_{grav} = \frac{-6\cdot67\times10^{-11}\times6\cdot4\times10^{23}\times2500}{(3390\times10^{3}+280\times10^{3})}$

$$= -2\cdot91\times10^{10}\ \text{J}$$

$\Delta E = -2\cdot91\times10^{10} - -3\cdot15\times10^{10} = \underline{\underline{2\cdot4\times10^{9}\ \text{J}}}$

Q2 d) $$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\therefore \quad v^2 = \frac{GM}{r} \quad \& \quad v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 6.4 \times 10^{23}}{3390 \times 10^3 + 280 \times 10^3}} = 3411 \text{ ms}^{-1}$$

$$E_K = \frac{mv^2}{2} = \frac{2500 \times 3411^2}{2} = \underline{\underline{1.45 \times 10^{10} \text{ J}}}$$

e) $$E_{TOTAL} = 2.4 \times 10^9 + 1.45 \times 10^{10} = \underline{\underline{1.69 \times 10^{10} \text{ J}}}$$

$$\% \text{ Kinetic} = \frac{1.45 \times 10^{10}}{1.69 \times 10^{10}} \times 100 = \underline{\underline{86\%}}$$

$$\% \text{ gravitational} = 100 - 86 = \underline{\underline{14\%}}$$

(3)

$$T = 24 \times 3600 + 40 \times 60 = \underline{88800\,s}$$

Q2 f) $\frac{mv^2}{r} = \frac{GMm}{r^2} \quad \therefore \quad v^2 = \frac{GM}{r}$

$$T = \frac{2\pi r}{v} \quad \therefore \quad v^2 = \left(\frac{2\pi r}{T}\right)^2$$

$$\therefore \frac{GM}{r} = \frac{4\pi^2 r^2}{T^2} \quad \therefore \quad r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{6.67\times10^{-11} \times 6.4\times10^{23} \times 88800^2}{4\pi^2}}$$

$$r = 2.043 \times 10^7\,m$$

Altitude = r - radius of mars

$= 2.043\times10^7 - 3390\times10^3$

$= 1.70\times10^7\,m = \underline{\underline{17000\ km}}$

Q3 a) Width of image = 22.0cm (on screen)
Width of Mars in image 16.5 cm

Fraction of Mars in image = $\frac{16.5}{22}$ = $\underline{0.75}$

$\therefore$ Number of pixels across Mars = 0.75×500
= 375 pixels

Resolution (size that 1 pixel represents) = $\frac{3390 \times 10^3}{375}$

= 9040 m = $\underline{\underline{9 \text{ km pixel}^{-1}}}$

It should be double this - 18 km/pixel I used the radius when I should have used the diameter!

b) $500 \times 500 \times 24 = 6 \times 10^6$ bits

$= 6 \times 10^6 / 8 = 7.5 \times 10^5$ bytes

= $\underline{\underline{750 \text{ kBytes or } 0.75 \text{ MBytes}}}$

c) Number of alternatives = 2^{24} = $\underline{\underline{16777216}}$

d) $6 \times 10^6 / 2.5 \times 60 = 4 \times 10^4 \text{ bit s}^{-1}$

$= \underline{\underline{5 \text{ kByte s}^{-1}}}$

Q3 e)

i) Increasing brightness could make it easier to see detail in dark areas of the image

ii) Increasing contrats could make it easier to see detail in uniform areas in the image where there is little variation in colour or brightness

iii) Noise reduction can remove any anomolous bright or dark spots making the image look smoother - but some detail may be lost.

iv) Edge detection will hilight places in the image where there is a sharp transition from one pixel value to another. For example where one type of surface changes to another at a boundary.

v) False colour could, like a contrast increase, highlight detail in areas of the image that seem to have little detail or variation in colour or shade.

Q4 a) $T^2 \propto r^3$

$\therefore T \propto \sqrt{r^3}$

radius increases by factor of $\dfrac{2.3 \times 10^{11}}{1.5 \times 10^{11}} = 1.53$

$\therefore$ T will increase by factor of $\sqrt{1.53^3} = 1.89$

$\therefore$ Period $= 365.25 \times 1.89 = \underline{\underline{690 \text{ Earth days}}}$

b) i) $v = \dfrac{2\pi r}{T} = \dfrac{2\pi \times 2.3 \times 10^{11}}{687 \times 24 \times 3600} = \underline{\underline{2.4 \times 10^4 \, ms^{-1}}}$

ii) $a = \dfrac{v^2}{r} = \dfrac{(2.4 \times 10^4)^2}{2.3 \times 10^{11}} = \underline{\underline{2.5 \times 10^{-3} \, ms^{-2}}}$

c) Min distance S E M $= 2.3 \times 10^{11} - 1.5 \times 10^{11}$

$= \underline{8 \times 10^{10} \, m}$

Time $= \dfrac{2 \times \text{dist}}{c} = \dfrac{2 \times 8 \times 10^{10}}{3 \times 10^8} = \underline{\underline{530 \text{ s}}}$ (8.9 min)

Max distance M S E $= 2.3 \times 10^{11} + 1.5 \times 10^{11}$

$= \underline{3.8 \times 10^{11} \, m}$

Time $= \dfrac{2 \times \text{dist}}{c} = \dfrac{2 \times 3.8 \times 10^{11}}{3 \times 10^8} = \underline{\underline{2500 \text{ s}}}$ (42 min)

5 a) $pV = \frac{1}{3}Nm\overline{c^2}$ & $pV = nRT = NkT$

$\therefore \quad \frac{1}{3}Nm\overline{c^2} = NkT$

$\therefore \quad \overline{c^2} \quad \frac{3kT}{m}$

$$C_{rms} = \sqrt{\overline{c^2}} = \sqrt{\frac{3kT}{m}}$$

b) i) Mass of CO_2 molecule $= \dfrac{4.4 \times 10^{-2}\ \text{kg mol}^{-1}}{6.02 \times 10^{23}\ \text{mol}^{-1}}$

$= \underline{7.31 \times 10^{-26}\ \text{kg}}$

$$C_{rms} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 210}{7.31 \times 10^{-26}}} = \underline{\underline{345\ \text{ms}^{-1}}}$$

ii) Mass of N_2 molecule $= \dfrac{2.8 \times 10^{-2}}{6.02 \times 10^{23}}$

$= 4.65 \times 10^{-26}\ \text{kg}$

$$C_{rms} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 210}{4.65 \times 10^{-26}}} = \underline{\underline{432\ \text{ms}^{-1}}}$$

5 c) $E_K = E_{grav}$

$$\frac{mv^2}{2} = \frac{GMm}{r} \quad \therefore \quad v^2 = \frac{2GM}{r}$$

$$V_{esc} = \sqrt{\frac{2GM}{r}}$$

d) $$V_{esc} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6.4 \times 10^{23}}{3390 \times 10^3}}$$

$$= \underline{\underline{5020 \text{ ms}^{-1}}}$$

e) At 210 K the root mean square speed of both CO_2 and N_2 is much less than the escape velocity. However this is a mean speed and some molecules are likely to exceed the escape velocity. It is more probable for N_2 as its C_{rms} is higher. This could be why Mars has an atmosphere of CO_2 and not N_2.

$$N_2 = \frac{432}{5020} \times 100 = 8.6\% \text{ of escape velocity}$$

$$CO_2 = \frac{345}{5020} \times 100 = 6.9\% \text{ of escape velocity}$$

⑨

Q6 a) Energy to escape $= -E_{grav} = \frac{GMm}{r}$

For CO_2 $\quad m = 4.4\times10^{-2}/6.02\times10^{23} = 7.31\times10^{-26}$ kg

$$E_{esc} = \frac{6.67\times10^{-11}\times 6.4\times10^{23}\times 7.31\times10^{-26}}{3390\times10^{3}}$$

$$= \underline{9.20\times10^{-19}\ J}$$

For N_2 $\quad m = 2.8\times10^{-2}/6.02\times10^{23} = 4.65\times10^{-26}$ kg

$$E_{esc} = \frac{6.67\times10^{-11}\times 6.4\times10^{23}\times 4.65\times10^{-26}}{3390\times10^{3}}$$

$$= \underline{5.85\times10^{-19}\ J}$$

$e^{-E/kT}$ Fraction for CO_2 $= e^{-9.2\times10^{-19}/1.38\times10^{-23}\times210}$

$$= \underline{e^{-317}}$$

Fraction for N_2 $= e^{-5.85\times10^{-19}/1.38\times10^{-23}\times210}$

$$= \underline{e^{-201}}$$

Ratio $= e^{-201}/e^{-317} = e^{-201--317} = e^{116}$

$$= \underline{\underline{2.4\times10^{50}}}$$

N_2 molecules are 10^{50} times more likely to escape Mars - so no N_2 on Mars.

Q6 b) Close to the surface $\Delta E_{grav} = mg\Delta h$

or $E_{grav} = mgh$ for short.

Fraction of molecules with energy $E = e^{-E/kT}$

$\therefore$ Fraction at height, h $= e^{-mgh/kT}$

since $pV = NkT$ $\quad p \propto N$ (number of molecules)

$$\therefore \ p = p_0 e^{-mgh/kT}$$

c) $p = 0.6 \times 10^3 \times e^{\left(\frac{-7.31 \times 10^{-26} \times 3.7 \times 22 \times 10^3}{1.38 \times 10^{-23} \times 210}\right)}$

$= \underline{\underline{77 \text{ Pa}}}$

Q7 a) $\text{Risk} = 20 \times 10^{-3} \times 3\% = \underline{\underline{0.06\%}}$

b) $\text{Dose} = 80 \times 10^{-3} \times 20 = 1.6\,Sv$

$\text{Risk} = 1.6 \times 3\% = 4.8\%$

$4.8\% \times 500 = \underline{\underline{24}}$

Q8 a) $E_K = \dfrac{mv^2}{2} \quad \therefore \quad v = \sqrt{\dfrac{2E_K}{m}}$

$$v = \sqrt{\frac{2 \times 10 \times 10^3 \times 1.6 \times 10^{-19}}{1.673 \times 10^{-27}}} = \underline{\underline{1.38 \times 10^6\,ms^{-1}}}$$

b) $\text{Angle} = \tan^{-1}\left(1/320\right) = \underline{\underline{0.179°}}$

OR $\approx \underline{\underline{1/320 \text{ radians}}}$

c) $F = qvB \quad \& \quad F = mv^2/r$

$\therefore \quad qvB = mv^2/r \quad \therefore \quad r = \dfrac{mv}{qB}$

$$r = \frac{1.673 \times 10^{-27} \times 1.38 \times 10^6}{1.6 \times 10^{-19} \times 2.0 \times 10^{-6}} = 7.2 \times 10^3\ m$$

$\theta = \dfrac{arc}{rad} = \quad \therefore \quad arc = \theta \times rad$

$= 1/320 \times 7.2 \times 10^3 = \underline{\underline{22.5m}}$