G494: Rise and Fall of the Clockwork Universe

Revision Notes

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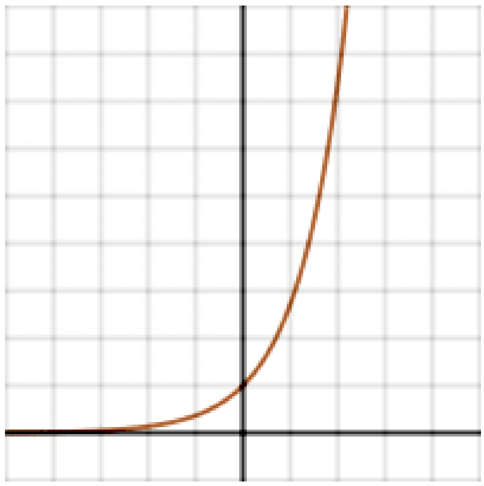


Graph of $x = ae^{bt}$, where b is positive.

Many things are governed by exponential relationships. The exponential relationships which we shall be dealing with are of the following form:

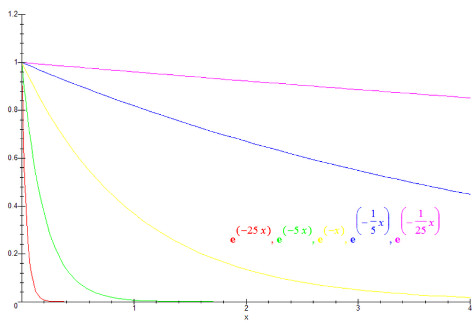
$x = ae^{bt}$,

where t is time, x is a variable, and a and b are constants. e is just a number, albeit a very special number. It is an irrational constant, like π. e is 2.71828182845904523536 to 20 decimal places. However, it is far easier just to find the e (or exp) button on your calculator.

The inverse function of e^t is the natural logarithm, denoted ln t:

$$\ln t = \log_e t$$

Growth and Decay



Graph of $x = e^{-bt}$, showing several different values of b.

When b is positive, an exponential function increases rapidly. This represents the growth of certain variables very well. When b is negative, an exponential function decreases, flattening out as it approaches the t axis. This represents the decay of certain variables.

Exponential Relationships in the Real World

An exponential relationship occurs when the rate of change of a variable depends on the value of the variable itself. You should memorise this definition, as well as understand it. Let us consider some examples:


A petri dish with bacteria growing on it.

Population Growth

Let us do some biology for a moment. Consider a Petri dish full of agar jelly (food for bacteria) with a few bacteria on it. These bacteria will reproduce, and so, as time goes by, the number of bacteria on the jelly will increase. However, each bacterium does not care about whether there are other bacteria around or not. It will continue making more bacteria at the same rate. Therefore, as the total number of bacteria increases, their rate of reproduction increases. This is an exponential relationship with a positive value of b.

Of course, this model is flawed since, in reality, the bacteria will eventually have eaten all the agar jelly, and so the relationship will stop being exponential.

Emptying Tank

If you fill a large tank with water, and make a hole in the bottom, at first, the water will flow out very fast. However, as the tank empties, the pressure of the water will decrease, and so the rate of flow will decrease. The rate of change of the amount of water in the tank depends on the amount of water in the tank. This is an exponential relationship with a negative value of b - it is an exponential decay.

Cooling

A hot object cools down faster than a warm object. So, as an object cools, the rate at which temperature 'flows' out of it into its surroundings will decrease. Newton expressed this as an exponential relationship (known as Newton's Law of Cooling):

$$T_t = T_{env} + (T_0 - T_{env})e^{-rt},$$

where T_t is the temperature at a time t, T_0 is the temperature at t = 0, T_{env} is the temperature of the environment around the cooling object, and r is a positive constant. Note that a here is equal to (T_0 - T_{env}) - but a is still a constant since T_0 and T_{env} are both constants. The '-' sign in front of the r shows us that this is an exponential decay - the temperature of the object is tending towards the temperature of the environment. The reason we add T_{env} is merely a result of the fact that we do not want the temperature to decay to 0 (in whatever unit of temperature we happen to be using). Instead, we want it to decay towards the temperature of the environment.

Mathematical Derivation

We have already said that an exponential relationship occurs when the rate of change of a variable depends on the value of the variable itself. If we translate this into algebra, we get the following:

$\frac{dx}{dt} = ax$, where a is a constant.

By separating the variables:

$$dx = axdt$$

$$\frac{1}{x}dx = adt$$

$$\int \frac{1}{x}dx = \int adt$$

$\ln x = at + c$ (where c is the constant of integration)

$x = e^{at+c} = e^{at}e^{c}$

If we let b = e^c (b is a constant, since e^c is a constant):

$x = be^{at}$

Questions

1. Simplify Newton's Law of Cooling for the case when I place a warm object in a large tank of water which is on the point of freezing. Measure temperature in °C.

2. What will the temperature of an object at 40 °C be after 30 seconds? (Take r=10^{-3} s^{-1}.)

3. A body is found in a library (as per Agatha Christie) at 8am. The temperature of the library is kept at a constant temperature of 20 °C for 10 minutes. During these 10 minutes, the body cools from 25 °C to 24 °C. The body temperature of a healthy human being is 36.8 °C. At what time was the person murdered?

4. Suppose for a moment that the number of pages on Wikibooks *p* can be modelled as an exponential relationship. Let the number of pages required on average to attract an editor be *a*, and the average number of new pages created by an editor each year be *z*. Derive an equation expressing *p* in terms of the time in years since Wikibooks was created *t*.

5. Wikibooks was created in mid-2003. How many pages should there have been 6 years later? (Take a = 20, z = 10 yr^{-1}.)

6. The actual number of pages in Wikibooks in mid-2009 was 35,148. What are the problems with this model? What problems may develop, say, by 2103?

/Worked Solutions/

A-level Physics (Advancing Physics)/Capacitors

A 10 μF capacitor

If you place two conducting plates near each other, with an insulator (known as a dielectric) in between, and you charge one plate positively and the other negatively, there will be an uniform electric field between them. Since:

$$E = \frac{V}{d},$$

as the distance between the two plates decreases, the voltage between them increases. This system is known as a capacitor - it has a capacitance for storing charge. The capacitance C of a capacitor is:

$$C = \frac{Q}{V},$$

where Q is the charge stored by the capacitor, and V is the potential difference between the plates. C is therefore the amount of charge stored on the capcitor per. unit potential difference. Capacitance is measured in farads (F). Just as 1 coulomb is a massive amount of charge, a 1F capacitor stores a lot of charge per. volt.

Any capacitor, unless it is physically altered, has a constant capacitance. If it is left uncharged, Q = 0, and so the potential difference across it is 0. If a DC power source is connected to the capacitor, we create a voltage across the capacitor, causing electrons to move around the circuit. This creates a charge on the capacitor equal to CV. If we then disconnect the power source, the charge remains there since it has nowhere to go. The potential difference across the capacitor causes the charge to 'want' to cross the dielectric, creating a spark. However, until the voltage between the plates reaches a certain level (the breakdown voltage of the capacitor), it cannot do this. So, the charge is stored.

If charge is stored, it can also be released by reconnecting the circuit. If we were to connect a wire of negligible resistance to both ends of the capacitor, all the charge would flow back to where it came from, and so the charge on the capacitor would again, almost instantaneously, be 0. If, however, we put a resistor (or another component with a resistance) in series with the capacitor, the flow of charge (current) is slowed, and so the charge on the capacitor does not become 0 instantly. Instead, we can use the charge to power a component, such as a camera flash.

Exponential Decay

Current is the rate of flow of charge. However, current is given by the formula:

$$I = \frac{V}{R}$$

But, in a capacitor, the voltage depends on the amount of charge left in the capacitor, and so the current is a function of the charge left on the capacitor. The rate of change of charge depends on the value of the charge itself. And so, we should expect to find an exponential relationship:

$$Q = Q_0 e^{-\frac{t}{RC}},$$

where R is the resistance of the resistor in series with the capacitor, Q is the charge on the capacitor at a time t and Q_0 was the charge on the capacitor at t = 0. Since Q = IΔt:

$$I\Delta t = I_0 \Delta t e^{-\frac{t}{RC}}$$

$$I = I_0 e^{-\frac{t}{RC}},$$

where I is the current flowing at a time t and I_0 was the initial current flowing at t = 0. Since V = IR:

$$V = V_0 e^{-\frac{t}{RC}}$$

The power being dissipated across the resitors in the circuit is IV, so:

$$P = I_0 V_0 e^{-\frac{t}{RC}} e^{-\frac{t}{RC}} = P_0 e^{-\frac{2t}{RC}}$$

Energy

The energy stored by a capacitor E is defined as:

$$E = \int_0^V Q\, dV$$

In other words, it is the area under a graph of charge against potential difference. Charge is proportional to potential difference (Q = CV), so the area under the graph is that of a triangle with base V and height Q. You can show this mathematically:

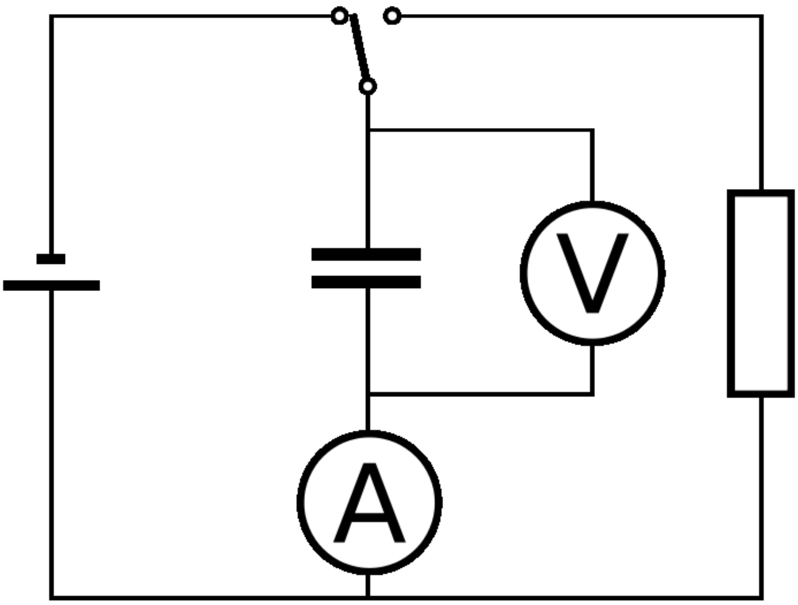
$$E = \int_0^V CV\, dV = C\left[\frac{V^2}{2}\right]_0^V = \frac{1}{2}CV^2$$

Since Q = CV:

$$E = \frac{1}{2}QV$$

Circuits

The circuit symbol for a capacitor is ⊣⊢ . A simple circuit with a capacitor in series with a resistor, an ideal ammeter (no resistance), and in parallel with an ideal voltmeter (infinite resistance) looks like the following:



In the position shown, the capacitor is charging. If the switch were put in the other position, the capacitor would be discharging exponentially through the resistor. In this circuit, the capacitor charges instantly since there is no resistance to slow it down. In reality there will be internal resistance in the battery, meaning that the capacitor charges exponentially.

If capacitors are placed in parallel, they act as one capacitor with a capacitance equal to the total of all the capacitances of all the individual capacitors. If capacitors are placed in series, the distances between the plates in each of them result in the capacitance of the imaginary resultant capacitor ΣC being given by:

$$\frac{1}{\Sigma C} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$$

Questions

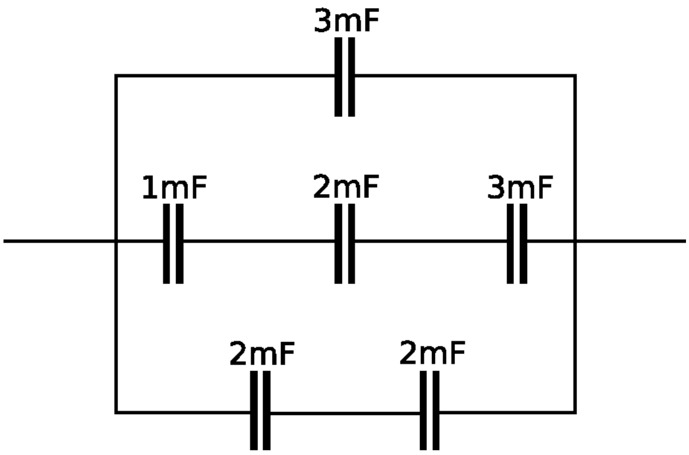
1. A 2 mF capacitor is connected to a 10V DC power supply. How much charge can be stored by the capacitor?

2. What is the highest possible energy stored by this capacitor?

3. The capacitor is placed in series with a 5Ω resistor and charged to capacity. How long would it take for the charge in the capacitor to be reduced to 1 mC?

4. After this time has elapsed, how much energy is stored in the capacitor?

5. What is the capacitance of the equivalent capacitor to the following network of capacitors?



/Worked Solutions/

A-level Physics (Advancing Physics)/Radioactive Decay

Decay Constant

We can model radioactive decay by assuming that the probability that any one nucleus out of N nuclei decays in any one second is a constant λ. λ is known as the decay constant, and is measured in s^{-1} (technically the same as Hz, but it is a probability, not a frequency, so we use s^{-1}).

Activity

As our N nuclei decay, the number of nuclei decreases. The activity of the N nuclei we have left is, on average, the probability that any one nucleus will decay per. unit time multiplied by the number of nuclei. If we have 200 nuclei, and the decay constant is 0.5, we would expect, on average, 100 nuclei to decay in one second. This rate would decreases as time goes by. This gives us the following formula for the activity A of a radioactive sample:

$$A = -\frac{dN}{dt} = \lambda N$$

Activity is always positive, and is measured in becquerels (Bq). It is easy to see that the rate of change of the number of nuclei is $-A = -\lambda N$.

Decay

The solution of the differential equation for activity given above is an exponential relationship:

$$N = N_0 e^{-\lambda t},$$

where N is the number of nuclei present at a time t, and N_0 is the number of nuclei present at time t=0. You can define t=0 to be any point in time you like, provided you are consistent. Since $A = \lambda N$ and therefore $A_0 = \lambda N_0$:

$$A = A_0 e^{-\lambda t},$$

where A is the activity of the sample at a time t, and A_0 is the activity at time t=0.

Questions

1 mole = 6.02 x 10^{23} atoms

1u = 1.66 x 10^{-27}kg

1. Americium-241 has a decay constant of 5.07 x $10^{-11}s^{-1}$. What is the activity of 1 mole of americium-241?

2. How many g of lead-212 (λ = 18.2μs^{-1}) are required to create an activity of 0.8 x 10^{18}Bq?

3. How long does it take for 2kg of lead-212 to decay to 1.5kg of lead-212?

4. Where does the missing 0.5kg go?

5. Some americium-241 has an activity of 3kBq. What is its activity after 10 years?

6. This model of radioactive decay is similar to taking some dice, rolling them once per. second, and removing the dice which roll a one or a two. What is the decay constant of the dice?

7. If you started out with 10 dice, how many dice would you have left after 10s? What is the problem with this model of radioactive decay?

/Worked Solutions/

A-level Physics (Advancing Physics)/Half-lives

The half life of something that is decaying exponentially is the time taken for the value of a decaying variable to halve.

Half Life of a Radioisotope

The most common use of half-lives is in radioactive decay. The activity is given by the equation:

$$A_t = A_0 e^{-\lambda t}$$

At $t=t_{½}$, A_t = ½A_0, so:

$$\frac{A_0}{2} = A_0 e^{-\lambda t_{\frac{1}{2}}} = \frac{A_0}{e^{\lambda t_{\frac{1}{2}}}}$$

$$2 = e^{\lambda t_{\frac{1}{2}}}$$

$$\ln 2 = \lambda t_{\frac{1}{2}}$$

Therefore:

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

It is important to note that the half-life is completely unrelated to the variable which is decaying. At the end of the half-life, all decaying variables will have halved. This also means that you can start at any point in the decay, with any value of any decaying variable, and the time taken for the value of that variable to halve from that time will be the half-life.

Half-Life of a Capacitor

You can also use this formula for other forms of decay simply by replacing the decay constant λ with the constant that was in front of the t in the exponential relationship. So, for the charge on a capacitor, given by the relationship:

$$Q_t = Q_0 e^{\frac{-t}{RC}}$$

So, substitute:

$$\lambda = \frac{1}{RC}$$

Therefore, the half-life of a capacitor is given by:

$$t_{\frac{1}{2}} = RC \ln 2$$

Time Constant of a Capacitor

However, when dealing with capacitors, it is more common to use the time constant, commonly denoted τ, where:

$$\tau = RC = \frac{t_{\frac{1}{2}}}{\ln 2}$$

At t = τ:

$$Q_t = Q_0 e^{\frac{-RC}{RC}} = \frac{Q_0}{e}$$

So, the time constant of a capacitor can be defined as the time taken for the charge, current or voltage from the capacitor to decay to the reciprocal of e (36.8%) of the original charge, current or voltage.

Questions

1. Radon-222 has a decay constant of $2.1\mu s^{-1}$. What is its half-life?

2. Uranium-238 has a half-life of 4.5 billion years. How long will it take for a 5 gram sample of U-238 to decay to contain 1.25 grams of U-238?

3. How long will it be until it contains 0.5 grams of U-238?

4. Tritium, a radioisotope of Hydrogen, decays into Helium-3. After 1 year, 94.5% is left. What is the half-life of tritium (H-3)?

5. A large capacitor has capacitance 0.5F. It is placed in series with a 5Ω resistor and contains 5C of charge. What is its time constant?

6. How long will it take for the charge in the capacitor to reach 0.677C? ($0.677 = \frac{5}{e^2}$)

/Worked Solutions/

Gravity

A-level Physics (Advancing Physics)/Gravitational Forces

Gravity is a force. Any object with mass exerts a gravitational force on any other object with mass. The force exerted by an object is proportional to its mass. However, this force is also inversely proportional to the distance between the objects squared. This means that, if two objects are twice as far away, the forces they exert on each other are four times smaller.

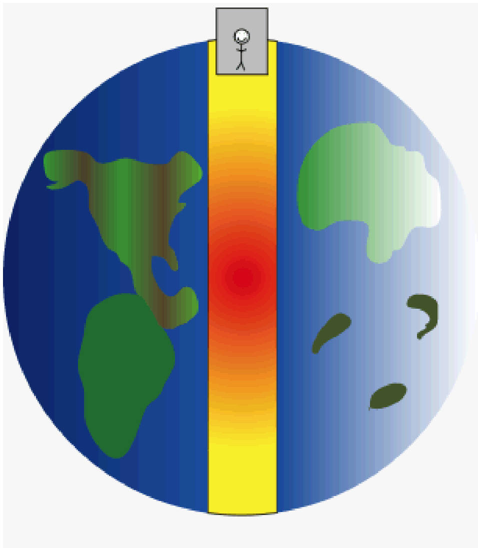
The gravitational force exerted by a sphere of mass M on another sphere of mass m is given by the following formula:

$$F_{grav} = \frac{-GMm}{r^2},$$

where r is the distance between the spheres, and G is a constant. Experiments have shown that G = 6.67 x 10^{-11} Nm^2kg^{-2}. The minus sign indicates that the force acts in the opposite direction to the radius. Radius is measured outwards from the centre, whereas gravity is an attractive force.

Gravitational Force Inside an Object

Inside a roughly spherical object (such as the Earth), it can be proved geometrically that the effects of the gravitational force resulting from all the mass outside a radius at which an object is located can be ignored, since it all cancels itself out. So, the only mass we need to consider is the mass inside the radius at which the object is located. The density of an object ρ is given by the following equation:



A lift acting under gravity in a lift shaft going through the centre of the Earth.

$$\rho = \frac{M}{V},$$

where M is mass, and V is volume. Therefore:

$$M = \rho V$$

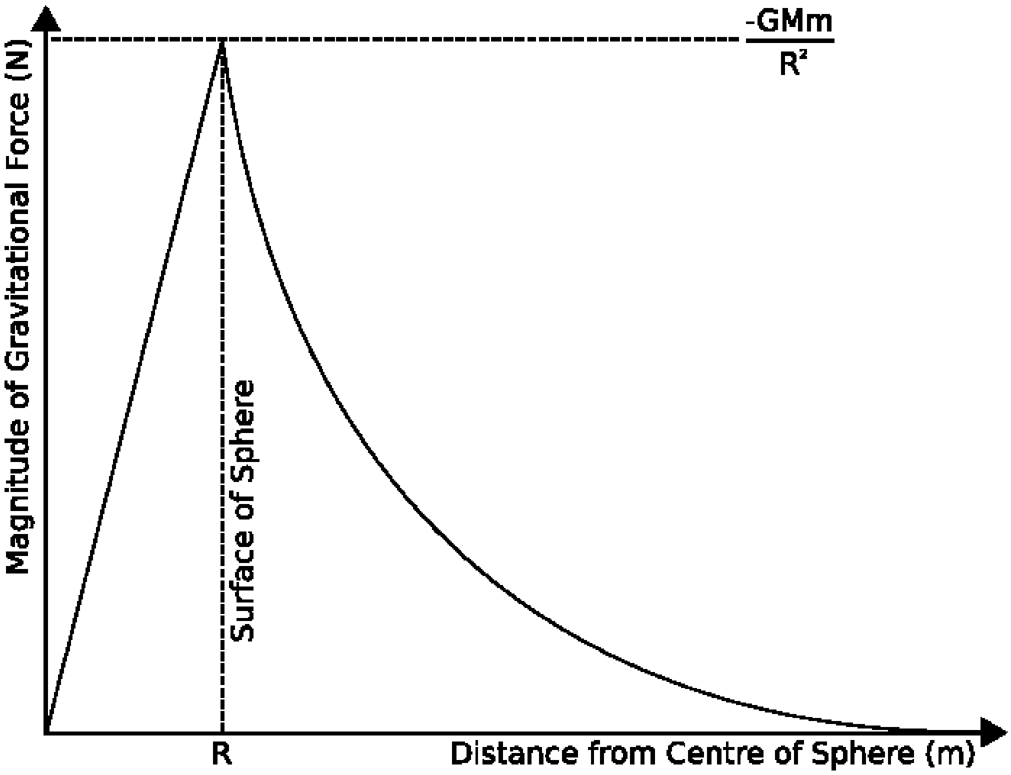
If we substitute the volume of a sphere for V:

$$M = \frac{4}{3}\pi\rho r^3$$

And if we substitute this mass into the formula for gravitational force given above:

$$F_{grav} = \frac{-Gm\frac{4}{3}\pi\rho r^3}{r^2} = -\frac{4}{3}\pi G\rho mr$$

In other words, inside a sphere of uniform mass, the gravitational force is directly proportional to the distance of an object from the centre of the sphere. Incidentally, this results in a simple harmonic oscillator such as the one on the right. This means that a graph of gravitational force against distance from the centre of a sphere looks like this:



Questions

1. Jupiter orbits the Sun at a radius of around 7.8 x 10^{11}m. The mass of Jupiter is 1.9 x 10^{27}kg, and the mass of the Sun is 2.0 x 10^{30}kg. What is the gravitational force acting on Jupiter? What is the gravitational force acting on the Sun?

2. The force exerted by the Sun on an object at a certain distance is 10^6N. The object travels half the distance to the Sun. What is the force exerted by the Sun on the object now?

3. How much gravitational force do two 1kg weights 5cm apart exert on each other?

4. The radius of the Earth is 6360km, and its mass is 5.97 x 10^{24}kg. What is the difference between the gravitational force on 1kg at the top of your body, and on 1kg at your head, and 1kg at your feet? (Assume that you are 2m tall.)

/Worked Solutions/

A-level Physics (Advancing Physics)/Gravitational Fields

The gravitational field, or gravitational field strength is the force exerted by gravity on an object per. unit mass of the object:

$$g = \frac{F_{grav}}{m}$$

As gravitational field strength is a measure of the force exerted on each unit of mass, its unit is Nkg^{-1}. If we consider a planet, Body A, the gravitational field strength experienced by another object, Body B, is given by:

$$g = \frac{\frac{-GMm}{r^2}}{m} = \frac{-GM}{r^2}$$

This is the total force exerted on Body B divided by the mass of Body B. Inside the planet, force is proportional to the distance from the centre, so the field is also proportional to distance.

Acceleration

Force is given by:

$$F = ma$$

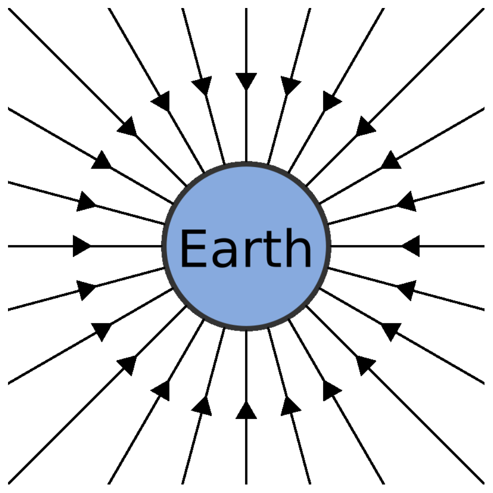
This means that:

$$g = \frac{ma}{m} = a$$

This shows that the gravitational field strength is also the acceleration due to gravity on any object. This acceleration is the same for any object, regardless of mass. When considering small heights above the Earth's surface, such as those in our day-to-day experiences, g remains roughly constant.

Field Lines

The gravitational field can be represented using field lines. These run in the direction that a mass would be accelerated in initially. The object will not necessarily fall along the field lines, but the acceleration will always be in the direction of the field lines. The closer the field lines are together, the denser the gravitational field.



Gravitational field lines around the Earth.

Questions

$G = 6.67 \times 10^{-11}\ m^3kg^{-1}s^{-2}$

1. A 15kg object has a weight of 8000N. What is the gravitational field strength at this point?

2. Draw a graph of gravitational field strength against distance.

3. What is the gravitational field strength of the Sun (mass 2×10^{30}kg) on the Earth (mass 6×10^{24}kg, mean orbital radius 15×10^{10}m)?

4. What is the difference in the acceleration due to gravity over a vertical distance d?

5. How far would one have to travel upwards from the Earth's surface to notice a $1Nkg^{-1}$ difference in gravitational field? (The Earth has a radius of 6400 km.)

/Worked Solutions/

A-level Physics (Advancing Physics)/Gravitational Potential Energy

If you throw a ball into the air, you give it kinetic energy. The ball then slows down because of the effect of the Earth's gravitational field on it. However, we know that energy cannot be created or destroyed. The kinetic energy you gave the ball is transformed into gravitational potential energy. The further away from the Earth you manage to throw the ball, the greater the potential there is for kinetic energy to be created on the way back down. However, for kinetic energy to be created, there must be an acceleration. If there is an acceleration, there must be a force.

You should already know that energy is the same as the work done to move something a distance Δx:

$$\Delta E_{grav} = F\Delta x$$

Work done is given by the force applied multiplied by the distance moved in the direction of the force. To move an object against gravity, the force applied upwards must equal the downwards force gravity exerts on the object, mg. So, if I move an object against gravity a distance Δx, the work done is given by:

$$\Delta E_{grav} = mg\Delta x$$

It is usual to call this x the height, so you will often see E_{grav}=mgh. The deltas are important. They mean that it doesn't matter which distance x I move the object across - I can decide the point at which gravitational potential energy is 0 in a way which makes the maths easy.

The difficulty with this simple formula is that g does not remain the same over large distances:

$$g = \frac{-GM}{r^2}$$

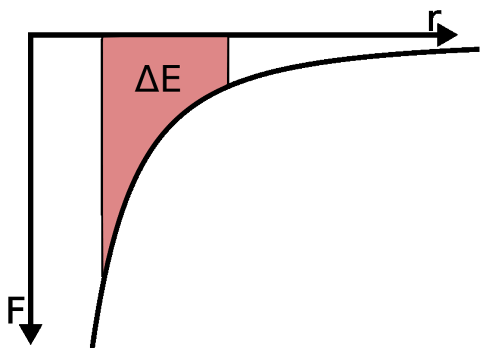
So, over a distance Δr, x becomes r and so:

$$E_{grav} = \frac{-GMmr}{r^2} = \frac{-GMm}{r}$$

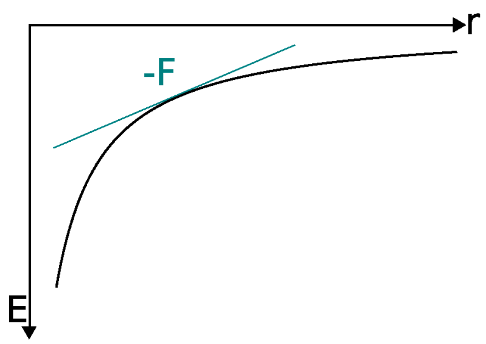
So, if you're dealing with gravitational potential energy over large distances, use this formula. If you're dealing with gravitational potential energy over short distances, such as with ramps on the Earth's surface, where g=9.81ms^{-2}, use E_{grav} = mgh.

Graphs

We have just done something sneaky. You probably didn't notice. Let's see what happens when we integrate the gravitational force F with respect to r between r and ∞:



The area under a graph of gravitational force against distance is the gravitational potential energy.



The gradient of a graph of gravitational potential energy against distance is the magnitude of the force.

$$\int_r^\infty \frac{-GMm}{r^2} dr = \left[\frac{GMm}{r}\right]_r^\infty = \frac{GMm}{\infty} - \frac{GMm}{r}$$

Since dividing anything by infinity gets you practically 0:

$$\int_r^\infty F dr = -\frac{GMm}{r} = E_{grav}$$

And therefore:

$$\frac{dE_{grav}}{dr} = F$$

So, if you have a graph of gravitational potential energy against radius, the gradient of the graph is the gravitational force. If you have a graph of gravitational force against radius, the area under the graph between any point and the F-axis is the gravitational potential energy at this point. The area under the graph between any two points is the difference in gravitational potential energy between them.

Questions

1. A ball rolls down a 3m-high smooth ramp. What speed does it have at the bottom?

2. In an otherwise empty universe, two planets of mass 10^{25} kg are 10^{12} m apart. What are their speeds when they collide?

3. What is the least work a 2000kg car must do to drive up a 100m hill?

4. How does the speed of a planet in an elliptical orbit change as it nears its star?

/Worked Solutions/

A-level Physics (Advancing Physics)/Gravitational Potential

Gravitational potential is the gravitational potential energy given to objects per. unit mass:

$$V_{grav} = \frac{E_{grav}}{m} = \frac{-\frac{GMm}{r}}{m} = \frac{-GM}{r}$$

Over short distances, gravitational potential energy is given by:

$$E_{grav} \approx mg\Delta h$$

So, over short distances, gravitational potential is equal to gΔh. Gravitational potential is measured in Jkg^{-1}.

Equipotentials

On a field diagram, lines can be drawn which, like contours on a map, show all the points which have the same gravitational potential. These lines are known as equipotentials. Equipotentials are always perpendicular to field lines, and get closer together as the field strength increases, and the density of field lines increases. Over short distances, equipotentials are evenly spaced.

Summary of Gravity

You should now know (if you did the gravity section in the right order) about four attributes of gravitational fields: force, field strength, potential energy and potential. These can be summarised by the following table:

Force $F_{grav} = \frac{GMm}{r^2}$	→ *integrate* → *with respect to r*	**Potential Energy** $E_{grav} = \frac{-GMm}{r}$
	↓ *per. unit mass* ↓	
Field Strength $g = \frac{GM}{r^2}$	→ *integrate* → *with respect to r*	**Potential** $V_{grav} = \frac{-GM}{r}$

Questions

$G = 6.67 \times 10^{-11}\ m^3kg^{-1}s^{-2}$

$g = 9.81\ ms^{-2}$

1. What is the gravitational potential at the Earth's surface? (mass of Earth = 5.97×10^{24} kg,radius of Earth = 6371 km)

2. Taking the Earth's surface as $V_{grav} = 0$, what is the gravitational potential 2m above the ground?

3. A 0.2kg firework reaches a gravitational potential relative to the ground of $500Jkg^{-1}$. If the firework is 30% efficient, how much energy was expended to get there?

4. Express gravitational potential in terms of gravitational force.

5. Draw the equipotentials and field lines surrounding the Earth.

/Worked Solutions/

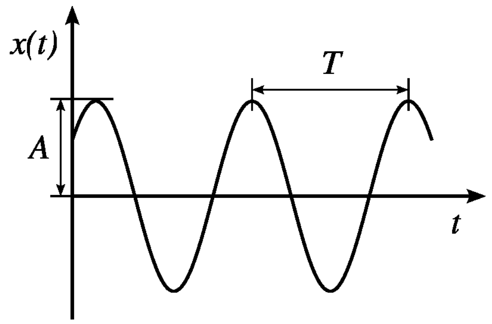
Mechanics

A-level Physics (Advancing Physics)/Simple Harmonic Motion

Simple harmonic motion occurs when the force on an object is proportional and in the opposite direction to the displacement of the object. Examples include masses on springs and pendula, which 'bounce' back and forth repeatedly. Mathematically, this can be written:

$$F = -kx,$$

where F is force, x is displacement, and k is a positive constant. This is exactly the same as Hooke's Law, which states that the force F on an object at the end of a spring equals -kx, where k is the spring constant. Since F = ma, and acceleration is the second derivative of displacement with respect to time t:



Graph of displacement against time in simple harmonic motion.

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = \frac{-kx}{m}$$

The solution of this second order differential equation is:

$$x = A\cos\omega t,$$

where A is the maximum displacement, and ω is the 'angular velocity' of the object. The derivation is given here, since it will seem very scary to those who haven't met complex numbers before. It should be noted that this solution, if given different starting conditions, becomes:

$$x = A\sin\omega t,$$

Angular Velocity

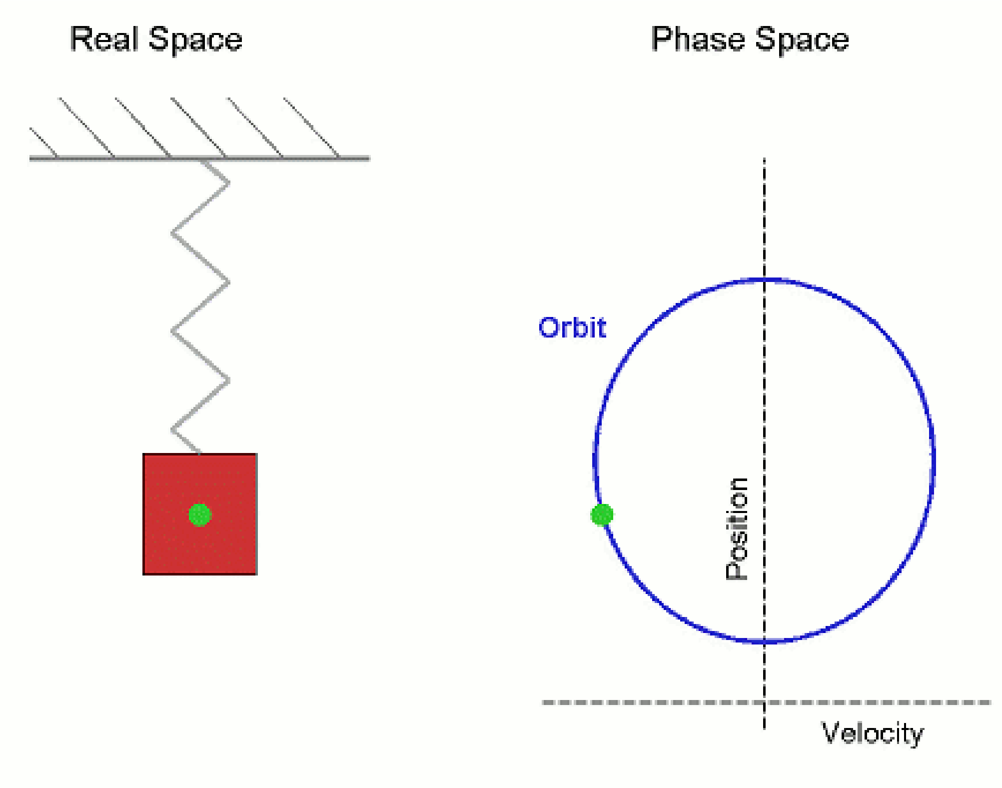
Angular velocity in circular motion is the rate of change of angle. It is measured in radians per. second. Since 2π radians is equivalent to one complete rotation in time period T:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

If we substitute this into the equation for displacement in simple harmonic motion:

$$x = A\cos 2\pi f t$$

The reason the equation includes angular velocity is that simple harmonic motion is very similar to circular motion. If you look at an object going round in a circle side-on, it looks exactly like simple harmonic motion. We have already noted that a mass on a spring undergoes simple harmonic motion. The following diagram shows the similarity between circular motion and simple harmonic motion:



Time Period

The time period of an oscillation is the time taken to repeat the pattern of motion once. In general:

$$T = \frac{2\pi}{\omega}$$

However, depending on the type of oscillation, the value of ω changes. For a mass on a spring:

$$\omega = \sqrt{\frac{k}{m}}$$

For a pendulum:

$$\omega = \sqrt{\frac{g}{l}},$$

where g is the gravitational field strength, and l is the length of the string. By substitution, we may gain the following table:

Type of Oscillation	Spring	Pendulum
Angular Velocity	$\sqrt{\frac{k}{m}}$	$\sqrt{\frac{g}{l}}$
Time Period	$2\pi\sqrt{\frac{m}{k}}$	$2\pi\sqrt{\frac{l}{g}}$

Velocity and Acceleration

The displacement of a simple harmonic oscillator is:

$$x = A\cos\omega t$$

Velocity is the rate of change of displacement, so:

$$v = \frac{dx}{dt} = -A\omega\sin\omega t$$

Acceleration is the rate of change of velocity, so:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -A\omega^2 cos\omega t = -\omega^2 x$$

Questions

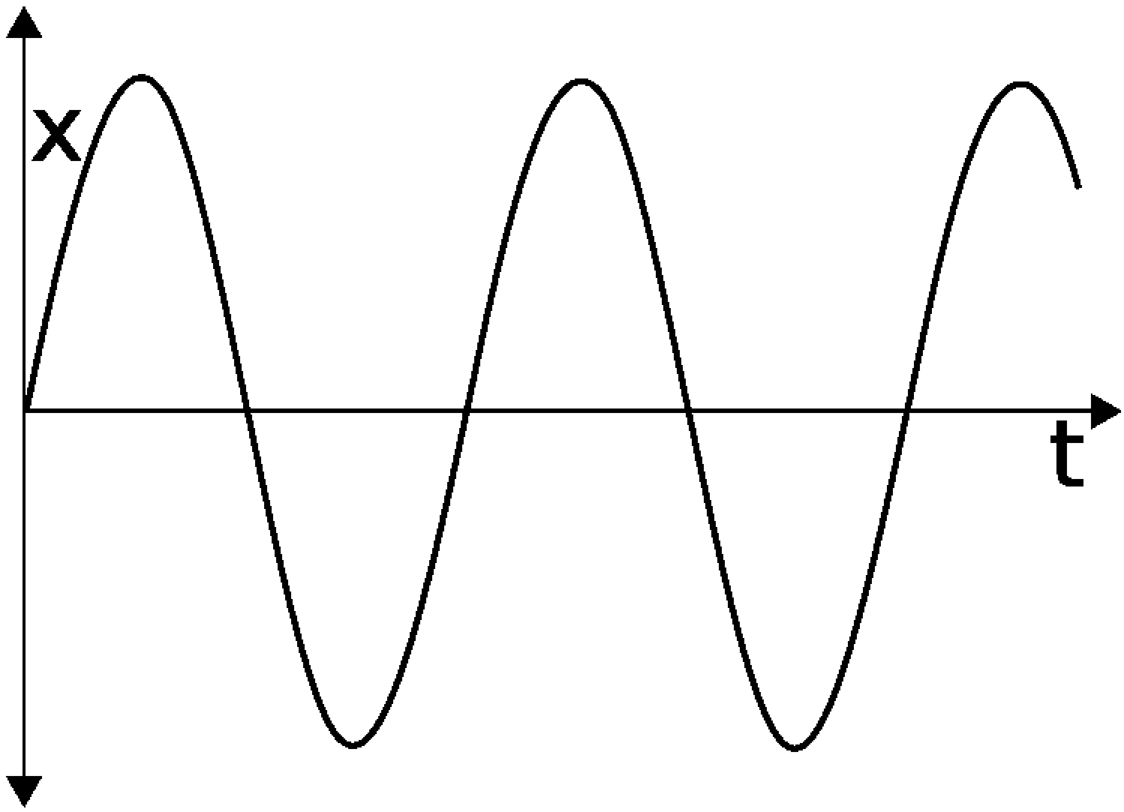
1. A 10N weight extends a spring by 5cm. Another 10N weight is added, and the spring extends another 5cm. What is the spring constant of the spring?

2. The spring is taken into outer space, and is stretched 10cm with the two weights attached. What is the time period of its oscillation?

3. What force is acting on the spring after 1 second? In what direction?

4. A pendulum oscillates with a frequency of 0.5 Hz. What is the length of the pendulum?

5. The following graph shows the displacement of a simple harmonic oscillator. Draw graphs of its velocity, momentum, acceleration and the force acting on it.



6. A pendulum can only be modelled as a simple harmonic oscillator if the angle over which it oscillates is small. Why is this?

/Worked Solutions/

A-level Physics (Advancing Physics)/Energy in Simple Harmonic Motion

A mass oscillating on a spring in a gravity-free vacuum has two sorts of energy: kinetic energy and elastic (potential) energy. Kinetic energy is given by:

$$E_k = \frac{1}{2}mv^2$$

Elastic energy, or elastic potential energy, is given by:

$$E_e = \frac{1}{2}kx^2$$

So, the total energy stored by the oscillator is:

$$\Sigma E = \frac{1}{2}(mv^2 + kx^2)$$

This total energy is constant. However, the proportions of this energy which are kinetic and elastic change over time, since v and x change with time. If we give a spring a displacement, it has no kinetic energy, and a certain amount of elastic energy. If we let it go, that elastic energy is all converted into kinetic energy, and so, when the mass reaches its initial position, it has no elastic energy, and all the elastic energy it did have has been converted into kinetic energy. As the mass continues to travel, it is slowed by the spring, and so the kinetic energy is converted back into elastic energy - the same amount of elastic energy as it started out. The nature of the energy oscillates back and forth, but the total energy is constant.

If the mass is oscillating vertically in a gravitational field, the situation gets more complicated since the spring now has gravitational potential energy, elastic potential energy and kinetic energy. However, it turns out (if you do the maths) that the total energy is still constant, although the equilibrium position will be lower.

Questions

1. A 10g mass causes a spring to extend 5cm. How much energy is stored by the spring?

2. A 500g mass on a spring (k=100) is extended by 0.2m, and begins to oscillate in an otherwise empty universe. What is the maximum velocity which it reaches?

3. Another 500g mass on another spring in another otherwise empty universe is extended by 0.5m, and begins to oscillate. If it reaches a maximum velocity of 15ms^{-1}, what is the spring constant of the spring?

4. Draw graphs of the kinetic and elastic energies of a mass on a spring (ignoring gravity).

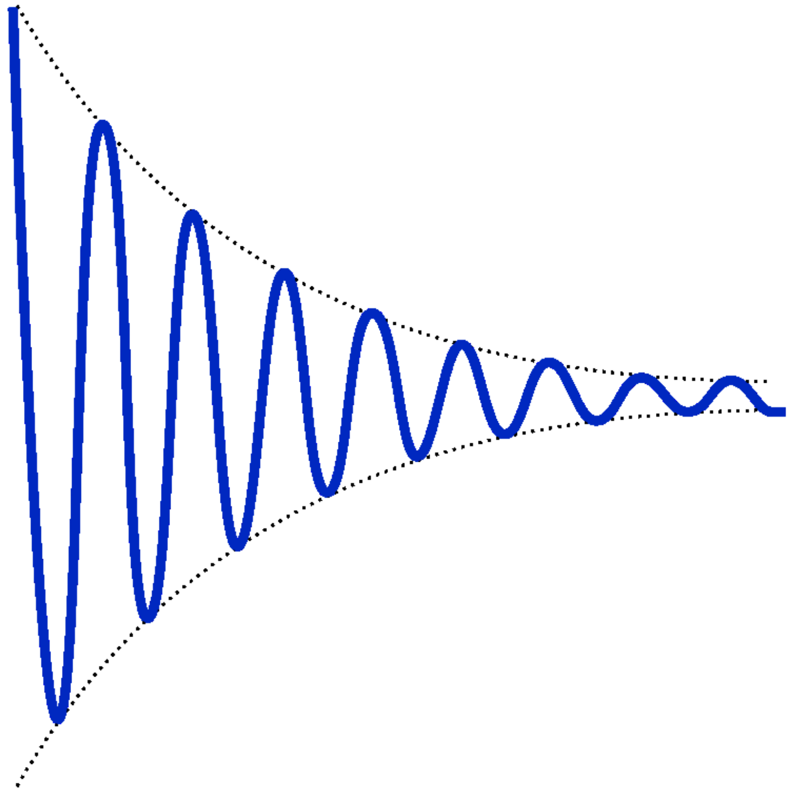
5. Use the trigonometric formulae for x and v to derive an equation for the total energy stored by an oscillating mass on a spring, ignoring gravity and air resistance, which is constant with respect to time.

/Worked Solutions/

A-level Physics (Advancing Physics)/Damping

Previously, our mathematical models of simple harmonic motion have assumed that the energy stored by a simple harmonic oscillator is constant. In reality, of course, resistive forces slow an oscillator down, transferring its energy to its surroundings. A pendulum will lose energy by moving the air. In addition to this, the motion of a mass on a spring will cause the spring to heat up, 'losing' the energy. This process is known as damping.

The principle effect of damping is to reduce the amplitude of an oscillation, not to change its frequency. So, the graph of the amplitude of a normal damped oscillation might look like the following:



Critical Damping

Critical damping occurs when a system is designed to return an oscillator to its equilibrium position in the least time possible. A critically damped oscillator, when damped, ceases to oscillate, and returns to its equilibrium position, where it stops moving. An example is the door closer. Normally, the door would swing back and forth, being damped by friction in the hinges, and air resistance. The door closer forces the door to stop swinging, and shut immediately. When closed, the door is in its equilibrium position.

Questions

1. Draw a graph of displacement for a critically damped oscillation.

2. How would you critically damp an oscillating pendulum?

3. How would you damp an oscillating pendulum using only a weighted polystyrene block?

4. What would the displacement graph look like for this oscillation, before and after damping began?

5. The graph above is an exponentially damped oscillation. If the displacement of the undamped oscillation is given by sin ωt, what is an approximate equation for the damped oscillation, in terms of a constant k which describes the degree to which the oscillation is damped?

/Worked Solutions/

A-level Physics (Advancing Physics)/Resonance

Resonance occurs when an oscillating system is driven (made to oscillate from an outside source) at a frequency which is the same as its own natural frequency. All oscillating systems require some form of an elastic force and a mass e.g. a mass at the end of a spring.

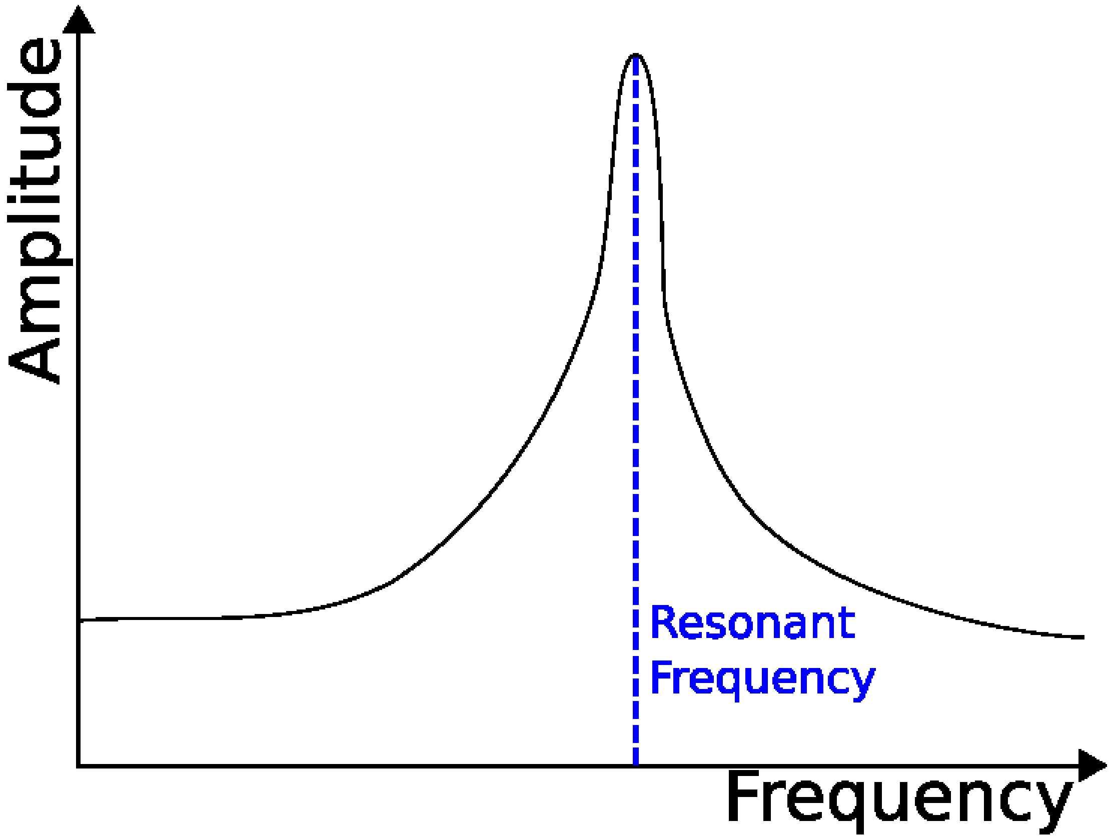
All oscillators have a natural frequency. If you have a mass on a spring, and give it an amplitude, it will resonate at a frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

This frequency is independent of the amplitude you give the oscillator to start with. It is the natural frequency of the oscillator. If you keep giving the oscillator amplitude at this frequency, it will not change the frequency of the oscillation. But, you are still doing work. This energy must go somewhere. The only place it can go is into additional kinetic and gravitational potential energy in the oscillation. If you force an oscillation at its resonant frequency, you add significantly to its amplitude.

Put simply, resonance occurs when the driving frequency of an oscillation matches the natural frequency, giving rise to large amplitudes.

If you were to force an oscillation at a range of frequencies, and measure the amplitude at each, the graph would look something like the following:



There are many types of oscillators, and so practically everything has a resonant frequency. This can be used, or can result in damage if the resonant frequency is not known.

Reading

Instead of doing questions this time, read the following articles on Wikipedia about these different types:

Tacoma Narrows Bridge

Resonance in Water Molecules (Microwave Ovens)

"No Highway" - a novel with a plot that uses things suspiciously similar to resonance.

Millenium Bridge

A-level Physics (Advancing Physics)/Conservation of Momentum

Momentum is the product of the mass of an object and its velocity. It is usually denoted p:

$p = mv$,

where m is mass, and v is velocity. The total momentum in a closed system is always conserved. This fact is useful, since it allows us to calculate velocities and masses in collisions.

Collisions

Let us consider a basic example: a ball of mass M collides with velocity u with a stationary ball of mass m. The stationary ball has no momentum before the collision, and the moving ball has momentum Mu. This must equal the momentum of both balls after the collision. If we let their velocities be v_1 and v_2:

$$Mu = Mv_1 + mv_2$$

At this point, we would need to know one of the velocities afterwards in order to calculate the other.

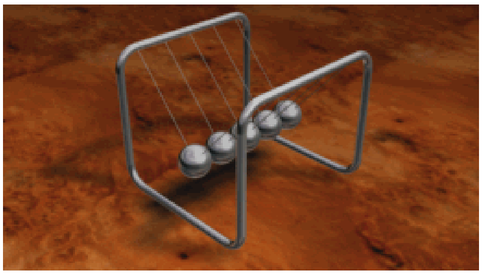
Alternatively, we could have one ball of mass M colliding with another ball of mass m, with both balls moving in opposite directions with velocities u_1 and u_2 respectively. If we define the direction of motion of the ball with mass M as the positive direction:

$$Mu_1 - mu_2 = Mv_1 + mv_2$$

We do not need to worry about the signs on the right-hand side: they will take care of themselves. If one of our velocities turns out to be negative, we know that it is in the opposite direction to u_1.

Elasticity

Although momentum within a closed system is always conserved, kinetic energy does not have to be. If kinetic energy is conserved in a collision, then it is known as a perfectly (or totally) elastic collision. If it is not conserved, then the collision is inelastic. If the colliding particles stick together, then a totally inelastic collision has occurred. This does not necessarily mean that the particles have stopped. In a totally inelastic collision, the two particles become one, giving the equation:



The collisions in a Newton's cradle are almost perfectly elastic.

$$Mu_1 + mu_2 = (M + m)v$$

$$v = \frac{Mu_1 + mu_2}{M + m}$$

Explosions

In an explosion, two particles which are stuck together are no longer stuck together, and so gain separate velocities:

$$(M+m)u = Mv_1 + mv_2$$

Questions

1. A ball of mass 0.5kg collides with a stationary ball of 0.6kg at a velocity of 3ms^{-1}. If the stationary ball moves off at a speed of 2ms^{-1}, what is the new velocity of the first ball?

2. Two balls are moving in opposite directions with velocities 5ms^{-1} and 10ms^{-1}. They collide, and move off in opposite directions with new velocities of 7.5ms^{-1} each. If the mass of the first ball was 1.25kg, what is the mass of the second ball?

3. A totally elastic collision occurs between two balls of equal mass. One of the balls is stationary. What happens?

4. A particle explodes to become two particles with masses 1kg and 2kg. The 1kg particle moves with velocity 45ms^{-1}. With what velocity does the other particle move?

5. A 3kg ball moving at 3ms^{-1} collides with a 5kg ball moving at -5ms^{-1}. The collision is perfectly elastic. What are the new velocities of the balls?

6. A ball collides with a wall, and rebounds at the same velocity. Why doesn't the wall move?

/Worked Solutions/

A-level Physics (Advancing Physics)/Forces and Impulse in Collisions

You should already know that the force exerted on an object is proportional to its acceleration. The constant of proportionality is known as the mass of the object:

$$F = ma$$

In the case of a collision, for one of the particles in the collision, the acceleration is simply the difference between its velocity before the collision (u) and its velocity after the collision (v) per unit. time:

$$F = \frac{m(v-u)}{\Delta t} = \frac{mv - mu}{\Delta t}$$

So, force is the rate of change of momentum. The quantity on top is known as the impulse of the collision, measured in Ns; Δ t is the length of time it took for the collision to take place. So, the impulse I is given by:

$$I = \Delta p = mv - mu = F\Delta t$$

In a collision where a certain change in momentum (impulse) occurs, a force is exerted. If the collision time is small, a larger force is exerted. If the collision time is long, a smaller force is exerted. If you have a graph of force against time, impulse is the area under the graph, since:

$$I = \int F \, dt$$

The impulse on one particle in a simple collision is the negative impulse on the other particle.

Questions

1. Escape velocity from the Earth is 11.2kms^{-1}. How much impulse must be exerted on a 47000kg payload to get it to travel away from the Earth?

2. Two billiard balls, of mass 10g, collide. One is moving at 5ms^{-1}, and the other at 2ms^{-1}. After the collision, the first billiard ball is moving backwards at 4ms^{-1}. The collision takes 1 ms. What force was exerted on this ball?

3. What impulse and force were exerted on the second ball?

4. A 60kg spacewalker uses a jet of gas to exert an impulse of 10Ns. How many times would he have to do this to reach a speed of 1 ms^{-1} from stationary?

5. A 5kg bowling ball collides with a stationary tennis ball of mass 0.1kg at 3ms^{-1}, slowing to 2.5ms^{-1}. It exerts a force of 100N on the ball. How long did the collision take?

/Worked Solutions/

A-level Physics (Advancing Physics)/Rockets, Hoses and Machine Guns

We have already seen that force is the rate of change of momentum. This applies to continuous flows of momentum as well as to collisions:

Apollo 15 launches itself to the moon by means of the change in momentum of its fuel.

$$F = \frac{dp}{dt} = v\frac{dm}{dt}$$

If I have a machine gun, explosions give the bullets of mass m momentum, causing them to move at a velocity v. This occurs several times each second - the momentum of the bullets is changing, and so there is a roughly continuous force acting on them. Momentum, of course, must be conserved. This results in a change in the momentum of the gun each time it fires a bullet. Overall, this results in a roughly continuous force on the gun which is equal and opposite to the force acting on the bullets.

If I have a tank of water and a hose, with a pump, and I pump the water out of the tank, a similar thing occurs - a force pushes me away from the direction of flow of the water. This force is equal to the flow rate (in kgs^{-1}) of the water multiplied by its velocity. Bear in mind that 1 litre of water has a mass of about 1kg.

Rockets work on this principle - they pump out fuel, causing it to gain momentum. This results in a thrust on the rocket. When designing propulsion systems for rockets, the aim is to give the fuel as high a velocity per. unit mass as possible in order to make the system fuel-efficient, and to get a high enough change in momentum.

Questions

1. A machine gun fires 300 5g bullets per. minute at $800ms^{-1}$. What force is exerted on the gun?

2. 1 litre of water is pumped out of a tank in 5 seconds through a hose. If a 2N force is exerted on the tank, at what speed does the water leave the hose?

3. If the hose were connected to the mains, what problems would there be with the above formula?

4. The thrust of the first stage of a Saturn V rocket is 34 MN, using 131000kg of solid fuel in 168 seconds. At what velocity does the fuel leave the tank?

5. Escape velocity from the Earth is $11kms^{-1}$. What is the velocity of the rocket after the first stage is used up, if the total mass of the rocket is 3×10^6 kg? How does this compare to escape velocity?

/Worked Solutions/

A-level Physics (Advancing Physics)/Circular Motion

Very rarely, things move in circles. Some planets move in roughly circular orbits. A conker on a string might move around my head in a circle. A car turning a corner might, briefly, move along the arc of a circle. The key thing to note about circular motion is that there is no force pulling outwards from the circle, and there is no force pulling the moving object tangential to the circle. Centrifugal force does not exist. There is only one force acting in circular motion, which is known as centripetal force. It always acts towards the centre of the circle. The object does not follow a circular path because two forces are balanced. Instead, the centripetal force accelerates the object with a constant magnitude in an ever-changing direction. The object has a velocity, and will continue moving with this velocity unless acted on by the centripetal force, which is perpetually adding velocity towards the centre of the circle.

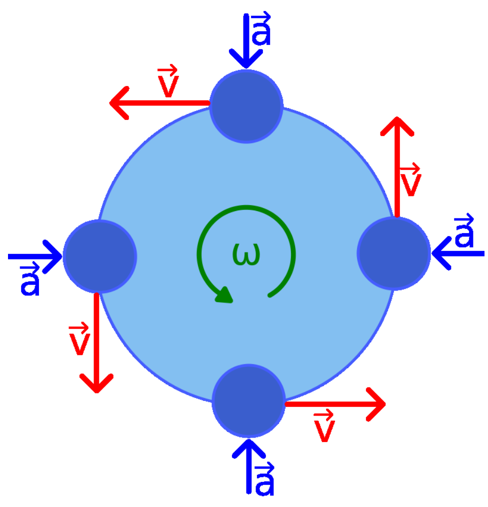


Diagram showing the key variables involved in circular motion.

If you were to subject a stationary object to the centripetal force, it would simply fall. If you gave it a little bit of velocity, it would still fall, but it would not land directly beneath its starting position. If you kept increasing the velocity and dropping it, there would come a point when it would land infinitely far away - it would go into orbit. The relationship between this 'magic' velocity and the magnitude of the centripetal force is as follows:

$$F = \frac{mv^2}{r},$$

where m is the mass of the object in circular motion, v is the magnitude of its velocity, and r is the distance from the centre of the circle to the object. Since F=ma, the centripetal acceleration is:

$$a = \frac{v^2}{r}$$

The centripetal force may manifest itself as many things: the tension in a string, friction, gravity or even an electric or magnetic field. In all these cases we can equate the equation for centripetal force with the equation for the force it really is.

Angular Velocity

Velocity is the rate of change of displacement. Angular velocity is the rate of change of angle, commonly denoted ω and measured in radians per. second:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

In circular motion:

$$\omega = \frac{2\pi}{T} = 2\pi f,$$

where T is the time for one revolution and f is the frequency of rotation. However:

$$v = \frac{2\pi r}{T}$$

$$\frac{v}{r} = \frac{2\pi}{T}$$

Therefore, the relationship between velocity and angular velocity is:

$$\omega = \frac{v}{r}$$

If we substitute this into the formula for centripetal acceleration:

$$a = \frac{(\omega r)^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r$$

Questions

1. A tennis ball of mass 10g is attached to the end of a 0.75m string and is swung in a circle around someone's head at a frequency of 1.5Hz. What is the tension in the string?

2. A planet orbits a star in a circle. Its year is 100 days, and the distance from the star to the planet is 70 Gm from the star. What is the mass of the star?

3. A 2000kg car turns a corner, which is the arc of a circle, at 20kmh^{-1}. The centripetal force due to friction is 1.5 times the weight of the car. What is the radius of the corner?

4. Using the formulae for centripetal acceleration and gravitational field strength, and the definition of angular velocity, derive an equation linking the orbital period of a planet to the radius of its orbit.

/Worked Solutions/

Astrophysics

A-level Physics (Advancing Physics)/Radar and Triangulation

Radar and triangulation are two relatively easy methods of measuring the distance to some celestial objects. Radar can also be used to measure the velocity of a celestial object relative to us.

Radar

Essentially, radar is a system which uses a radio pulse to measure the distance to an object. The pulse is transmitted, reflected by the object, and then received at the site of the transmitter. The time taken for all this to happen is measured. This can be used to determine the distance to a planet or even the velocity of a spaceship.

Distance

The speed of electromagnetic waves (c) is constant in a vacuum: 3×10^8 ms^{-1}. If we fire a pulse of radio waves to a planet within the Solar System, we know that:

$$d = ct$$

where d is the distance to the planet, and t is the time taken for the pulse to get there. However, the pulse has to get both there and back, so:

$$2d = ct$$

$$d = \frac{ct}{2}$$

where d is the distance to the planet, and t is the time taken for the pulse to return.

Velocity

The velocity of an object can be found by firing two radar pulses at an object at different times. Two distances are measured. When asked to calculate the relative velocity of an object in this way, use the following method:

1. Calculate the distance to the object at both times:

$$d_1 = \frac{c\Delta t_1}{2}$$

$$d_2 = \frac{c\Delta t_2}{2}$$

2. Calculate the distance the object has travelled between the two pulses. This is the difference between the two distances previously calculated:

$$\Delta d = d_2 - d_1$$

3. Calculate the time between the transmission (or reception, but not both) of the two pulses:

$$\Delta t = t_2 - t_1$$

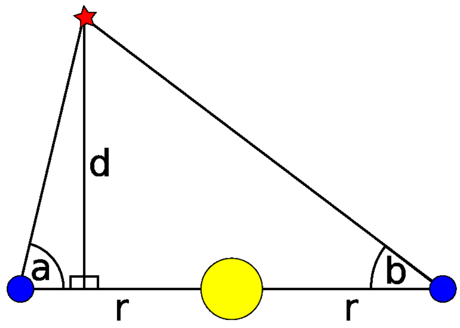
4. Divide the distance calculated in step 2 by the time calculated in step 3 to find the average velocity of the object between the transmission of the two pulses:

$$v = \frac{\Delta d}{\Delta t}$$

Triangulation

We know that the Earth is, on average, about 150 Gm away from the Sun. If we measure the angle between the vertical and the light from a nearby star 6 months apart (ie. on opposite sides of the Sun), we can approximate the distance from the Solar System to the star.

Let r be the radius of the Earth's orbit (assumed to be constant for simplicity's sake), a and b be the angles to the star (from the horizontal) when the Earth is on either side of the Sun, and let d be the perpendicular distance from the plane of the Earth's orbit to the star, as shown in the diagram on the right. By simple trigonometry:



The distance from the solar system to a relatively nearby celestial object can be found using triangulation.

$$2r = \frac{d}{\tan a} + \frac{d}{\tan b} = \frac{d(\tan a + \tan b)}{\tan a \tan b}$$

Therefore:

$$d = \frac{2r \tan a \tan b}{\tan a + \tan b}$$

Questions

1. A radar pulse takes 8 minutes to travel to Venus and back. How far away is Venus at this time?

2. Why can't a radar pulse be used to measure the distance to the Sun?

3. Radar is used to measure the velocity of a spacecraft travelling between the Earth and the Moon. Use the following data to measure this velocity:

Pulse	Transmission Time	Reception Time
1	3:26:45.31213	3:26:45.51213
2	3:26:46.32742	3:26:46.52785

4. The angles between the horizontal and a star are measured at midnight on January 1 as 89.99980° and at midnight on July 1 as 89.99982°. How far away is the star?

5. Why can't triangulation be used to measure the distance to another galaxy?

/Worked Solutions/

A-level Physics (Advancing Physics)/Large Units

The distances in space are so large that we need some very large units to describe them with.

Light Years

A light year is the distance that light travels in one year. The velocity of light is constant ($3 \times 10^8 ms^{-1}$), so 1 light year is:

$$3 \times 10^8 \times 365.24 \times 24 \times 60 \times 60 \approx 9.46 \times 10^{15} \text{ m}$$

Light seconds, light minutes, light hours and light days are less commonly used, but may be calculated in a similar fashion.

Astronomical Units

1 astronomical unit (denoted AU) is the mean average distance from the Earth to the Sun. This is approximately 150×10^9m.

Parsecs

Degrees can be divided into minutes and seconds. There are 60 minutes in a degree, and 60 seconds in a minute. This means that 1 second is $^1/_{3600}$ of a degree. A degree is denoted °, a minute ' and a second ". The definition of a parsec uses a simplified form of triangulation. It assumes that the perpendicular to the plane of the Earth's orbit passes through the Sun and a celestial object. A parsec is the distance from the Sun to this celestial object if the angle between the perpendicular and the the line connecting the Earth to the celestial object. This gives us the following right-angled triangle (the distance from the Earth to the Sun is 1 AU):

$$\tan 1'' = \tan \left(\frac{1}{3600}^{\circ} \right) = 4.85 \times 10^{-6} = \frac{1}{\text{parsec}}$$

Therefore, a parsec is 206,265 AU.

Questions

1. What is one parsec in m?

2. Convert 3 light days into km.

3. Convert 5.5 parsecs into light years.

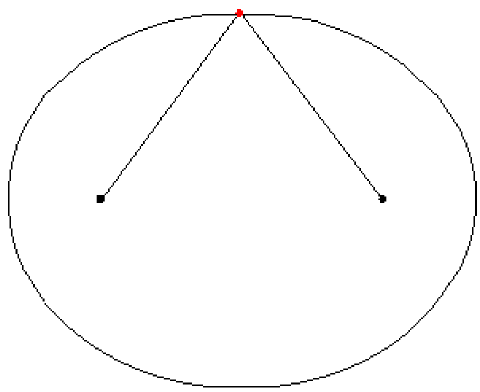
4. The difference in angle of a star on the perpendicular to the plane of the Earth's orbit which passes through the Sun when viewed from either side of the Earth's orbit is 0.1°. How far away is the star in parsecs?

/Worked Solutions/

A-level Physics (Advancing Physics)/Orbits

Planets orbit a sun. Theoretically, their orbit may be circular. This case is dealt with under circular motion.

In reality, planets orbit in ellipses. An ellipse is a shape which has two foci (singular 'focus'). The total of the distances from any point on an ellipse to its foci is constant. All orbits take an elliptical shape, with the sun as one of the foci. As the planet approaches its sun, its speed increases. This is because gravitational potential energy is being converted into kinetic energy.



The total of the distances from any point on an ellipse to its foci is constant.

A circle is an ellipse, in the special case when both foci are at the same point.

Kepler's Third Law

Kepler's third law states that:

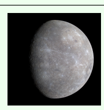
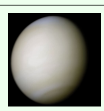
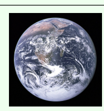
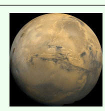
> "The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit."

Mathematically, for orbital period T and semi-major axis R:

$$T^2 \propto R^3$$

This result was derived for the special case of a circular orbit as the fourth circular motion problem. The semi-major axis is the distance from the centre of the ellipse (the midpoint of the foci) to either of the points on the edge of the ellipse closest to one of the foci.

Questions

Planet	Mercury	Venus	Earth	Mars
Picture				
Mean distance from Sun (km)	57,909,175	108,208,930	149,597,890	227,936,640
Orbital period (years)	0.2408467	0.61519726	1.0000174	1.8808476

1. The semi-major axis of an elliptical orbit can be approximated reasonably accurately by the mean distance of the planet for the Sun. How would you test, using the data in the table above, that the inner planets of the Solar System obey Kepler's Third Law?

2. Perform this test. Does Kepler's Third Law hold?

3. If $T^2 \alpha R^3$, express a constant C in terms of T and R.

4. Io, one of Jupiter's moons, has a mean orbital radius of 421600km, and a year of 1.77 Earth days. What is the value of C for Jupiter's moons?

5. Ganymede, another of Jupiter's moons, has a mean orbital radius of 1070400km. How long is its year?

/Worked Solutions/

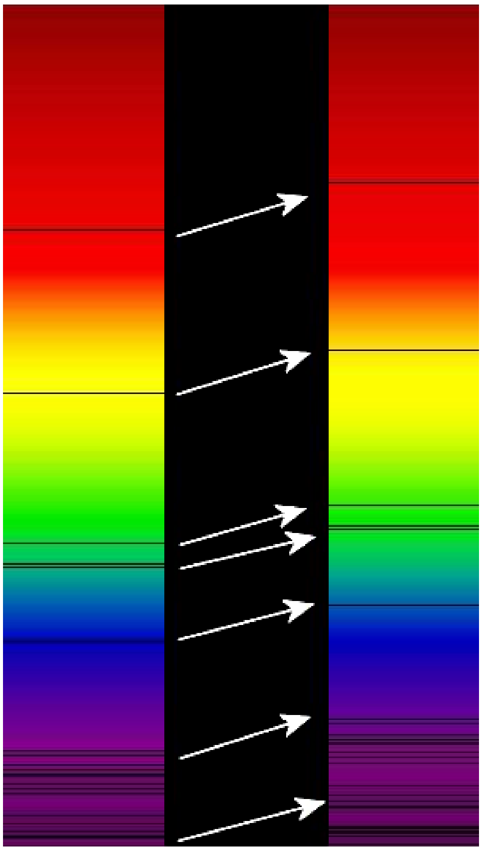
A-level Physics (Advancing Physics)/Doppler Effect

The Doppler effect is a change in the frequency of a wave which occurs if one is in a different frame of reference from the emitter of the wave. Relative to us, we observe such a change if an emitter of a wave is moving relative to us.

All waves travels in a medium. So, they have a velocity relative to this medium v. They also have a velocity relative to their source v_s and a velocity relative to the place where they are received v_r. The frequency at which they are received f is related to the frequency of transmission f_0 by the formula:

$$f = \left(\frac{v + v_r}{v + v_s}\right) f_0$$

The Doppler effect can be used to measure the velocity at which a star is moving away from or towards us by comparing the wavelength receive λ with the wavelength we would expect a star of that type to emit λ_0. Since the speed of light c is constant regardless of reference medium:



Redshift of spectral lines in the optical spectrum of a supercluster of distant galaxies (right), as compared to that of the Sun (left).

$$c = f\lambda = f_0\lambda_0$$

Therefore:

$$f = \frac{c}{\lambda} \text{ and } f_0 = \frac{c}{\lambda_0}$$

By substitution:

$$\frac{c}{\lambda} = \left(\frac{v+v_r}{v+v_s}\right)\frac{c}{\lambda_0}$$

$$\frac{1}{\lambda} = \left(\frac{v+v_r}{v+v_s}\right)\frac{1}{\lambda_0}$$

$$\lambda = \frac{\lambda_0(v+v_s)}{v+v_r}$$

In this case, v is the speed of light, so v = c. Relative to us, we are stationary, so $v_r = 0$. So:

$$\lambda = \frac{\lambda_0(c+v_s)}{c}$$

$$\frac{\lambda}{\lambda_0} = \frac{(c+v_s)}{c} = 1 + \frac{v_s}{c}$$

If we call the change in wavelength due to Doppler shift $\Delta\lambda$, we know that $\lambda = \lambda_0 + \Delta\lambda$. Therefore:

$$\frac{\lambda_0 + \Delta\lambda}{\lambda_0} = 1 + \frac{\Delta\lambda}{\lambda_0} = 1 + \frac{v_s}{c}$$

So, the important result you need to know is that:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v_s}{c} = z$$

This value is known as the red-shift of a star, denoted z. If z is positive, the star is moving away from us - the wavelength is shifted up towards the 'red' end of the electromagnetic spectrum. If z is negative, the star is moving towards us. This is known as blue shift. Note that we have assumed that v is much smaller than c. Otherwise, special relativity makes a significant difference to the formula.

Questions

1. M31 (the Andromeda galaxy) is approaching us at about 120kms^{-1}. What is its red-shift?

2. Some light from M31 reaches us with a wavelength of 590nm. What is its wavelength, relative to M31?

3. Some light has a wavelength, relative to M31, of 480nm. What is its wavelength, relative to us?

4. A quasar emits electromagnetic radiation at a wavelength of 121.6nm. If, relative to us, this wavelength is red-shifted 0.2nm, what is the velocity of recession of the quasar?

/Worked Solutions/

A-level Physics (Advancing Physics)/Big Bang Theory

Big Bang theory states that space-time began as a single point, and that, as time passed, space itself expanded.

Hubble's Law

Hubble's Law describes the expansion of the universe mathematically:

$$v = H_0 d\,,$$

where v is the velocity of recession of a celestial object, and d is the distance to the object. H_0 is the Hubble constant, where $H_0 = 70kms^{-1}Mpc^{-1}$. The '0' signifies that this is the Hubble constant now, not in the past or the future. This allows for the fact that the Hubble constant might be changing, but very slowly.

The Age of the Universe

Imagine a galaxy which flies out from the big bang at the speed of light (c). The distance it has travelled d is given by:

$$d = vt\,,$$

where t is the age of the universe, since the galaxy has been travelling since the beginning. If we substitute in Hubble's Law for v, we get:

$$d = H_0 dt$$

$$1 = H_0 t$$

$$t = \frac{1}{H_0}$$

So, the reciprocal of the Hubble constant is the age of the universe - but be careful with the units.

More Doppler Effect

We have already seen that red-shift z is given by:

$$z = \frac{\Delta\lambda}{\lambda_s} = \frac{v_s}{c},$$

where Δλ is the amount by which radiation is red-shifted from a celestial object, λs is the wavelength of the radiation relative to the celestial object, vs is the velocity of recession of the object, c is the speed of light, and v is much less than c. If \lambda is the wavelength of the radiation relative to us:

$$z = \frac{\lambda - \lambda_s}{\lambda_s} = \frac{\lambda}{\lambda_s} - 1$$

$$z + 1 = \frac{\lambda}{\lambda_s}$$

However, if it is actually space that is being stretched, then this is actually the ratio of the distances between us and the celestial object at two times: the time at which the radiation was emitted, and the time at which the radiation was received. We can apply this to any distance between any two stars:

$$\frac{R_{now}}{R_{then}} = z + 1$$

Evidence for the Big Bang

Red Shift

If we measure the red shift of celestial objects, we see that most of them are moving away from us - the light from them is red-shifted. This is not true of all celestial objects - the Andromeda galaxy, for example, is blue-shifted; it is moving towards us due to the gravitational attraction of the Milky Way. Some galaxies are partly red-shifted, and partly blue-shifted. This is due to their rotation - some parts of the galaxy are rotating towards us, while others are rotating away from us. However, the majority of celestial objects are moving away from us. If we extrapolate backwards, we find that the universe must have started at a single point. However, we are assuming that the universe has always expanded. Red shift provides evidence for a Big Bang, but does not prove it.

Cosmic Microwave Background Radiation

Models of the Big Bang show that, at the beginning of the universe, radiation of a relatively short wavelength would have been produced. Now, this radiation, due to the expansion of space, has been stretched - it has become microwave radiation. Cosmic microwave background radiation fits in extremely well with Big Bang theory, and so is strong evidence for it.

Questions

1. What is the Hubble Constant in s^{-1}?

2. How old is the universe?

3. What effect might gravity have had on this figure?

4. Polaris is 132pc away. What is its velocity of recession, according to Hubble's Law?

/Worked Solutions/

Temperature

A-level Physics (Advancing Physics)/Heat and Energy

Matter is made of particles. These particles are constantly moving. When we feel some matter, we feel something that we call 'heat'. This is just our impression of how fast the particles are moving. The higher the average speed of the particles, the hotter something is.

Note that there is a technical difference between 'heat' and 'temperature'. Heat is energy in transit, also known as work. Temperature is the internal energy of a substance. We feel heat, not temperature, for if if the energy did not move from an object, we would not be able to measure it.

If we had some matter which was made of stationary particles, then we would not be able to make the particles more stationary. The concept is meaningless. When matter is in this state, it is at the coldest temperature possible. We call this temperature 0°K. This corresponds to -273.15°C. If the temperature rises by 1°K, the temperature rises by 1°C. The only difference between the two scales is what temperature is defined as 0 - in Kelvin, 0 is absolute zero. In Celsius, 0 is the freezing point of water. In both scales, 1° is one hundredth of the difference in temperature between the freezing and boiling points of water.

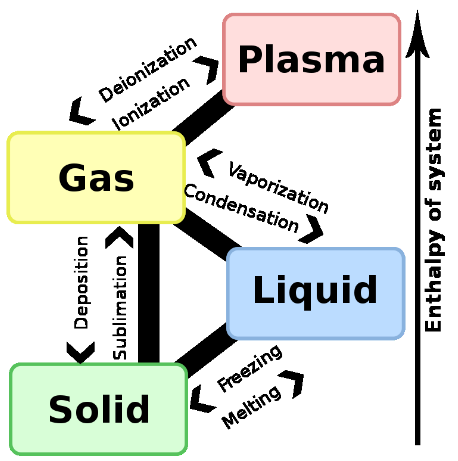
Some matter of temperature T consists of many particles. Their motion is essentially random - they are all moving at different speeds, and so they all have different kinetic energies. The temperature is related to the *average* energy per. particle E by the following *approximate* relationship:

$$E \approx kT,$$

where k is a constant known as the Boltzmann constant. k = 1.38 x $10^{-23}$$JK^{-1}$. T must always be measured in Kelvin.

Changes of State

Different substances change state at different temperatures. In other words, when the average energy per. particle reaches a certain level, the substance changes state. The situation complicates itself since, in order to change state, additional energy is required (or is given out). When liquid water reaches its boiling point, it will stay at its boiling point until it has all changed into water vapour since the energy being taken in is being used to change state, instead of to increase the temperature of the water. The average energy per. particle required to change state can be *approximated* using the formula above, where T is the temperature at which the substance changes state.



The relationships between the states of matter.

Activation Energy

Many things have an activation energy. In order for a chemical reaction to start, for example, the average energy per. particle must reach a certain level. However, most of the time, chemical reactions start at a lower average energy per. particle than the activation energy. This is because there is always a chance that some particles have the required activation energy, since the particles are moving at random. If the reaction is exothermic (this means that it gives out heat, raising the average energy per. particle), then, once one reaction has happened, more of the particles have the activation energy, and so the reaction accelerates until all the reagants are used up. The activation energy can be related to the temperature of the substance using the formula E=kT.

Questions

1. Carbon dioxide sublimes at 195°K. Roughly what energy per. particle does this correspond to?

2. A certain chemical reaction requires particles with mass of the order 10^{-26} to move, on average, at $10ms^{-1}$. Roughly what temperature does this correspond to?

3. The boiling point of water is 100°C. Roughly what energy per. particle does this correspond to?

4. Thermionic emission from copper requires around 5eV of energy per. particle. How hot will the wire be at this energy level?

/Worked Solutions/

A-level Physics (Advancing Physics)/Specific Heat Capacity

It takes energy to heat things up, since heat is work. If we heat a more massive thing up, it takes more work, because we have to give more particles, on average, an energy kT. Some substances require more work to heat up than others. This property is known as specific heat capacity. This gives us the formula:

$\Delta E = mc\Delta\theta$,

where ΔE is the work put in to heating something up (in J), m is the mass of the thing we are heating up (in kg), c is the specific heat capacity (in $Jkg^{-1}K^{-1}$), and Δθ is the difference in temperature due to the work done on the substance (in degrees Celsius or Kelvin).

It should be noted that the specific heat capacity changes slightly with temperature, and more than slightly when the material changes state. A table of the specific heat capacities of various substances is given below:

Substance	State	Temperature (°C)	Specific Heat Capacity ($kJkg^{-1}K^{-1}$)
Air	gas	23	1.01
Aluminium	solid	25	0.90
Animal (and human) tissue	mixed	25	3.5
Argon	gas	25	0.52
Copper	solid	25	0.39
Glass	solid	25	0.84
Helium	gas	25	5.19
Hydrogen	gas	25	14.3
Iron	solid	25	0.45
Lead	solid	25	0.13
Nitrogen	gas	25	1.04

Oxygen	gas	25	0.92
Uranium	solid	25	0.12
Water	solid	-10	2.05
Water	liquid	25	4.18
Water	gas	100	2.08

Questions

1. How much work would it take to heat 100kg of liquid water from 20°C to 36.8°C?

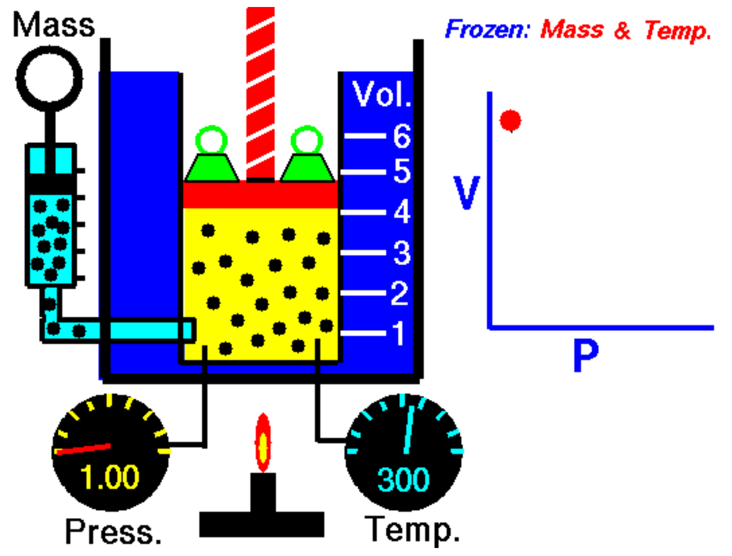
2. How much work would it take to heat a well-insulated room from 15°C to 21°C, if the room is a cube with side length 10m, and the density of the air is $1.2kgm^{-3}$?

3. A 10kg block of iron at 80°C is placed in the room above once it has reached 21°C. If the iron cools by 40°C, what is the new temperature of the room?

/Worked Solutions/

A-level Physics (Advancing Physics)/Ideal Gases

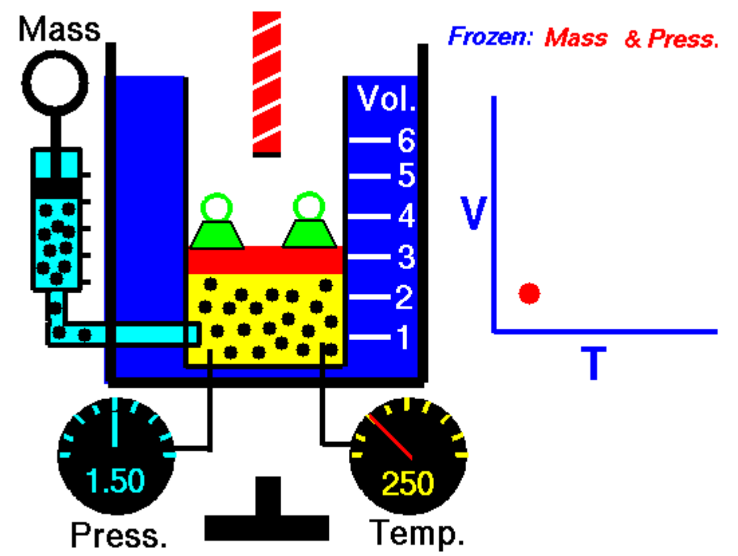
Real-world gases can be modelled as ideal gases. An ideal gas consists of lots of point particles moving at random, colliding with each other elastically. There are four simple laws which apply to an ideal gas which you need to know about:



An animation showing the relationship between pressure and volume when mass and temperature are held constant.

Boyle's Law

Boyle's Law states that the pressure of an ideal gas is inversely proportional to its volume, assuming that the mass and temperature of the gas remain constant. If I compress an ideal gas into half the space, the pressure on the outsides of the container will double. So:



An animation demonstrating the relationship between volume and temperature.

$$p \propto \frac{1}{V}$$

Charles' Law

Charles' Law states that the volume of an ideal gas is proportional to its temperature:

$$V \propto T$$

T must be measured in kelvin, where 1°K = 1°C, but 0°C = 273°K. If we double the temperature of a gas, the particles move around twice as much, and so the volume also doubles.

Amount Law

This law states that the pressure of an ideal gas is proportional to the amount of gas. If we have twice the number of gas particles N, then twice the pressure is exerted on the container they are in:

$$p \propto N$$

A mole is a number of particles. 1 mole = 6.02 x 10^{23} particles. So, the pressure of a gas is also proportional to the number of moles of gas present n:

$$p \propto n$$

Pressure Law

The pressure law states that the pressure of an ideal gas is proportional to its temperature. A gas at twice the temperature (in °K) exerts twice the pressure on the sides of a container which it is in:

$$p \propto T$$

These laws can be put together into larger formulae linking p, V, T and N.

Questions

1. I heat some argon from 250K to 300K. If the pressure of the gas at 250K is 0.1 MPa, what is its pressure after heating?

2. The argon is in a 0.5m long cylindrical tank with radius 10cm. What volume does it occupy?

3. The argon is then squeezed with a piston so that in only occupies 0.4m of the tank's length. What is its new pressure?

4. What is its new temperature?

5. 25% of the argon is sucked out. What is its pressure now?

/Worked Solutions/

A-level Physics (Advancing Physics)/Kinetic Theory

One formula which sums up a lot of the kinetic theory of an ideal gas is the following:

$$pV = \frac{1}{3}Nm\bar{c^2},$$

where p is the pressure of the gas, V is its volume, N is the number of molecules, m is the mass of each molecule, and $\bar{c^2}$ is the mean square speed of the molecules. If you knew the speeds of all the molecules, you could calculate the mean square speed by squaring each speed, and then taking the mean average of all the squared speeds.

Derivation

This formula can be derived from first principles by modelling the gas as a lot of particles colliding. The particles have a momentum p = mc. If we put them in a box of volume V and length l, the change in momentum when they hit the side of the box is:

$$\Delta p = m(c - (-c)) = 2mc$$

Every time the particle travels the length of the box (l) and back (another l), it hits the wall, so:

$$c = \frac{2l}{t},$$

where t is the time between collisions. Therefore:

$$t = \frac{2l}{c}$$

Each collision exerts a force on the wall. Force is the rate of change of momentum, so:

$$F = \frac{\Delta p}{\Delta t} = \frac{2mc}{\frac{2l}{c}} = \frac{2mc^2}{2l} = \frac{mc^2}{l}$$

However, we have got N particles all doing this, so the total force on the wall is given by:

$$F = \frac{Nm\bar{c^2}}{l}$$

The molecules all have different velocities, so we have to taken an average - the mean square speed. This force is the force in all three dimensions. The force in only one dimension is therefore:

$$F = \frac{Nm\bar{c^2}}{3l}$$

Pressure, by definition, is:

$$p = \frac{F}{A} = \frac{Nm\bar{c^2}}{3Al}$$

But area multiplied by length is volume, so:

$$p = \frac{Nm\bar{c^2}}{3V}$$

Therefore:

$$pV = \frac{1}{3}Nm\bar{c^2}$$

Questions

1. Five molecules are moving at speeds of 1,5,6,8, and 36ms^{-1}. What is their mean square speed?

2. What is the mass of one molecule of N_2 (atomic mass 14, 1u = 1.66 x 10^{-23}kg)?

3. Atmospheric pressure is 101,325 Pa. If one mole of Nitrogen takes up 2.3 m^3 at about 10°C, what is the mean square speed of the molecules in the air outside, assuming that the atmosphere is 100% nitrogen (in reality, it is only 78%)?

4. What is the average speed of a nitrogen molecule under the above conditions?

5. The particles in question 1 are duplicated 3000 times. If they have a completely unrealistic mass of 1g, what is their pressure when they are crammed into a cube with side length 0.5m?

/Worked Solutions/

A-level Physics (Advancing Physics)/Boltzmann Factor

Particles in a gas lose and gain energy at random due to collisions with each other. On average, over a large number of particles, the proportion of particles which have at least a certain amount of energy ε is constant. This is known as the Boltzmann factor. It is a value between 0 and 1. The Boltzmann factor is given by the formula:

$$\frac{n}{n_0} = e^{\frac{-\epsilon}{kT}},$$

where n is the number of particles with kinetic energy above an energy level ε, n_0 is the total number of particles in the gas, T is the temperature of the gas (in kelvin) and k is the Boltmann constant (1.38 x 10^{-23} JK^{-1}).

This energy could be any sort of energy that a particle can have - it could be gravitational potential energy, or kinetic energy, for example.

Derivation

In the atmosphere, particles are pulled downwards by gravity. They gain and lose gravitational potential energy (mgh) due to collisions with each other. First, let's consider a small chunk of the atmosphere. It has horizontal cross-sectional area A, height dh, molecular density (the number of molecules per. unit volume) n and all the molecules have mass m. Let the number of particles in the chunk be N.

$$n = \frac{N}{V} = \frac{N}{A\,dh}$$

Therefore:

$V = A\,dh$ (which makes sense, if you think about it)

By definition:

$$N = nV = nA\,dh$$

The total mass Σ m is the mass of one molecule (m) multiplied by the number of molecules (N):

$\Sigma m = mN = mnA\ dh$

Then work out the weight of the chunk:

$W = g\Sigma m = nmgA\ dh$

The downwards pressure P is force per. unit area, so:

$P = \frac{W}{A} = \frac{nmgA\ dh}{A} = nmg\ dh$

We know that, as we go up in the atmosphere, the pressure decreases. So, across our little chunk there is a difference in pressure dP given by:

$dP = -nmg\ dh$ (1)

We also know that:

$PV = NkT$

So:

$P = \frac{NkT}{V}$

But:

$n = \frac{N}{V}$

So, by substitution:

$P = nkT$

So, for our little chunk:

$dP = kT\ dn$ (2)

If we equate (1) and (2):

$dP = -nmg\ dh = kT\ dn$

Rearrange to get:

$\frac{dn}{dh} = \frac{-nmg}{kT}$

Integrate between the limits n_0 and n:

$h = \frac{-kT}{mg}\int_{n_0}^{n}\frac{1}{n}\ dn = \frac{-kT}{mg}\left[\ln n\right]_{n_0}^{n} = \frac{-kT}{mg}\left(\ln n - \ln n_0\right) = \frac{-kT}{mg}\ln\frac{n}{n_0}$

$ln\frac{n}{n_0} = \frac{-mgh}{kT}$

$\frac{n}{n_0} = e^{\frac{-mgh}{kT}}$

Since we are dealing with gravitational potential energy, ε = mgh, so:

$\frac{n}{n_0} = e^{\frac{-\epsilon}{kT}}$

Questions

1u = 1.66 x 10^{-27} kg

g = 9.81 ms^{-2}

1. A nitrogen molecule has a molecular mass of 28u. If the Earth's atmosphere is 100% nitrous, with a temperature of 18°C, what proportion of nitrogen molecules reach a height of 2km?

2. What proportion of the molecules in a box of hydrogen (molecular mass 2u) at 0°C have a velocity greater than 5ms^{-1}?

3. What is the temperature of the hydrogen if half of the hydrogen is moving at at least 10ms^{-1}?

4. Some ionised hydrogen (charge -1.6 x 10^{-19} C)is placed in an uniform electric field. The potential difference between the two plates is 20V, and they are 1m apart. What proportion of the molecules are at least 0.5m from the positive plate (ignoring gravity) at 350°K?

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