Answer **all** the questions.

SECTION A

1 Here is a list of units.

$kg\,m^{-1}\,s^{-2}$ $kg\,m\,s^{-1}$ $kg\,m^2\,s^{-1}$ $kg\,m^2\,s^{-2}$

(a) Which **one** is a correct unit for momentum?

$p = mv \rightarrow kg\,m\,s^{-1}$

$kg\,m\,s^{-1}$ [1]

(b) Which **one** is a correct unit for pressure?

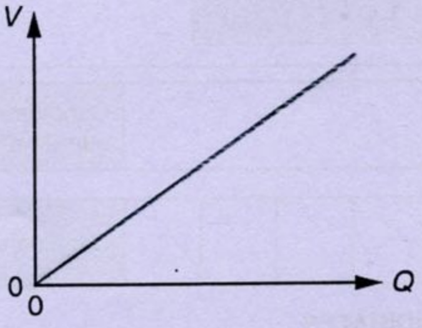
$p = F/A \quad F = ma \Rightarrow \frac{kg\,m\,s^{-2}}{m^2}$

$kg\,m^{-1}\,s^{-2}$ [1]

2 **(a)** Sketch a graph on the axes below to show how the potential difference V across a capacitor varies with the charge Q on its plates.

$C = \frac{Q}{V}$

$V = \frac{Q}{C} \quad V \propto Q$

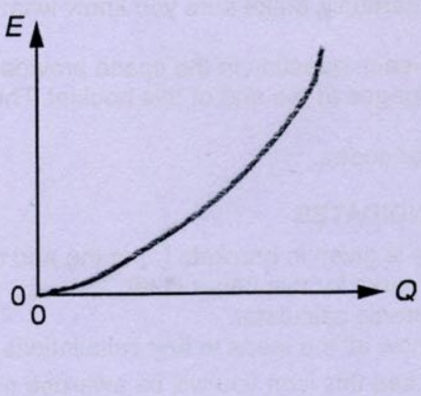


[1]

(b) Sketch a graph on the axes below to show how the energy E stored in the capacitor varies with the charge Q on its plates.

$E = \frac{1}{2}QV$

$= \frac{1}{2}Q^2/C$



[1]

3 This question is about the behaviour of a mass suspended by a spring from a vibrating support.

(a) Describe the condition required for the mass-spring system to go into **resonance**.

driving frequency $\approx$ natural frequency

[1]

(b) Damping is applied to the mass-spring system.

Describe and explain one effect of damping when the system goes into resonance.

Max amplitude reduced as damping removes energy from vibrating system

[2]

4 The atmosphere of Mars is mostly carbon dioxide at a mean temperature of –63 °C. = 210 K

Estimate the speed v of carbon dioxide molecules at this temperature.

mass of a carbon dioxide molecule = 7.3×10^{-26} kg

$k = 1.4 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

$E_K \approx kT = \frac{1}{2}mv^2$

$\therefore v = \sqrt{\frac{2kT}{m}} = 284\ \mathrm{ms^{-1}}$

OR $\cancel{N}kT = \frac{1}{3}\cancel{N}m\overline{c^2}$

$c = \sqrt{\overline{c^2}} = \sqrt{\frac{3kT}{m}} = 348\ \mathrm{ms^{-1}}$

v = ... $\mathrm{m\,s^{-1}}$ **[3]**

5 A student does an experiment to verify that momentum is conserved when a pair of trolleys on a track collide head-on.

Fig. 5.1 shows the trolleys on a level track approaching each other.

+ →

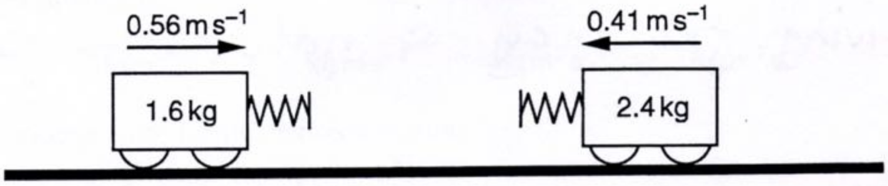


Fig. 5.1

Fig. 5.2 shows the situation **after** the trolleys collide.

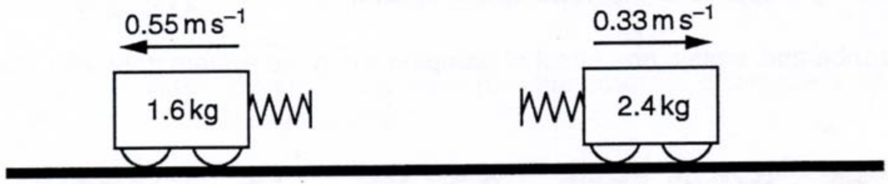


Fig. 5.2

Show that momentum is conserved in the collision.

$p_{initial} = +0.56 \times 1.6 + -0.41 \times 2.4 = -0.088$

$p_{final} = -0.55 \times 1.6 + +0.33 \times 2.4 = -0.088$ kgm

$p_i = p_f$

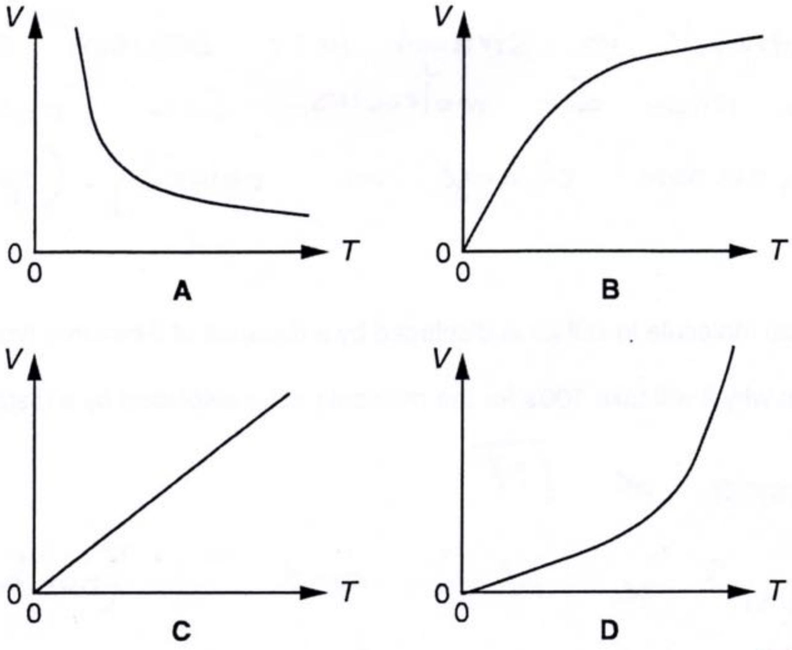
[2]

6 Which one of these four graphs (**A**, **B**, **C** or **D**) shows how the volume V of a fixed mass of gas at a constant pressure changes with kelvin temperature T?

$PV = nRT$

const

$P \propto T$



answer ....C.......... **[1]**

Question 7 begins on page 6

7 This question is about the random walk of a molecule through air.

(a) Explain why the molecule does a random walk.

It travels in straight lines between collisions with other air molecules. Each collision gives a random change in velocity. (speed & direction)

[2]

(b) A typical molecule in still air is displaced by a distance of 5 mm in a time of 1 s.

Explain why it will take 100 s for the molecule to be displaced by a distance of 50 mm.

distance $\propto \sqrt{N}$

$\therefore$ dist^2 $\propto$ N and N (number of collisions will be $\propto$ to time t.

$\therefore$ if time is $\times 100$ d is $\times 10$

[2]

8 The recessional velocity of a distant galaxy is measured to be 3.5×10^3 km s^{-1}.

Use the age of the Universe (14×10^9 years) to estimate the distance from Earth to this galaxy. State the assumption you have to make.

1 year = 3.2×10^7 s

Age in sec $= 14 \times 10^9 \times 3.2 \times 10^7 = 4.48 \times 10^{17}$ s

Distance travelled at 3.5×10^3 km s^{-1}

$= 4.48 \times 10^{17} \times 3.5 \times 10^3 = 1.6 \times 10^{21}$ km

Assumption is that v is constant.

distance = 1.6×10^{24} m [3]

[Section A Total: 20]

SECTION B

9 The asteroid belt between Mars and Jupiter contains a large number of rocks in circular orbit around the Sun.

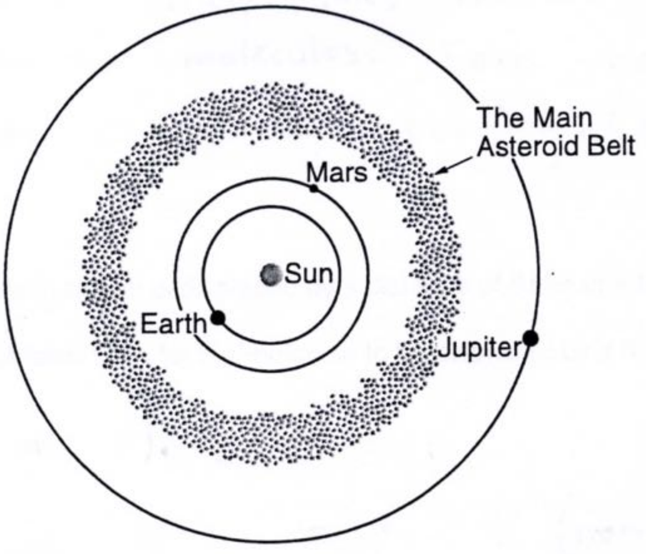


Fig. 9.1

(a) Show that the speed v of an asteroid of mass m in a circular orbit of radius r around the Sun of mass M is given by

$$v = \sqrt{\frac{GM}{r}}.$$

$F_{grav} = F_{centripetal} \quad \therefore \frac{GMm}{r^2} = \frac{mv^2}{r}$

$\therefore \frac{GM}{r} = v^2 \quad \therefore v = \sqrt{\frac{GM}{r}}$

[2]

(b) One particular asteroid in the asteroid belt is in a circular orbit of radius 3.6×10^{11} m.

(i) Show that it has a kinetic energy of about 10^{11} J.

mass of asteroid = 500 kg
mass of Sun = 2.0×10^{30} kg
$G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}

$E_K = \frac{mv^2}{2} = \frac{GMm}{2r} = \frac{6.7\times10^{-11} \times 2\times10^{30} \times 500}{2\times 3.6\times10^{11}}$

$= 9.3\times 10^{10}$ J

[2]

(ii) It is believed that collisions between asteroids can put them into elliptical orbits which cross the Earth's orbit. These asteroids may then collide with the Earth, causing widespread damage.
If a collision changes the direction of motion of the asteroid in **(i)** **without** changing its kinetic energy, calculate its speed v when it crosses the Earth's orbit.

radius of the Earth's orbit = 1.5×10^{11} m

It will gain KE as it loses E grav.

E_{grav} (initial) $= -\frac{GMm}{r} = -1.86 \times 10^{11}$ J

E_{grav} (final) $= \quad = -4.47 \times 10^{11}$ J

$\boxed{\Delta E_{grav} = -2.61 \times 10^{11} \text{ J}}$

E_K final $= \; = 9.3 \times 10^{10} + 2.61 \times 10^{11} = 3.54 \times 10^{11}$ J

$v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2 \times 3.54 \times 10^{11}}{500}}$

v = 3.76×10^4 m s^{-1} **[4]**

(c) Astronomers study many asteroids whose orbits approach the Earth's orbit.

Explain how the distance from Earth to a nearby asteroid can be measured using radar.

Your answer should state clearly the assumptions behind the method you describe.

send out radar pulse and time how long for echo to return.

dist $= \frac{\text{time}}{2} \times$ speed of light

ass = speed constant (c) & out and return journeys take same time **[3]**

[Total: 11]

10 This question is about heating the water in a swimming pool which has the cross-sectional shape shown in Fig. 10.1.

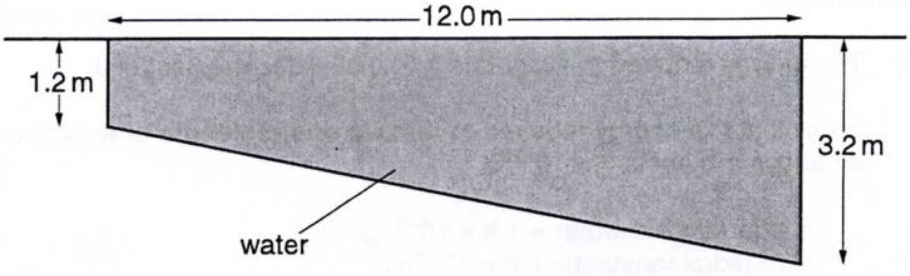


Fig. 10.1

(a) The pool has a constant width of 5.6 m.

Show that it contains about 1.5×10^5 kg of water.

density of water = $1.0 \times 10^3 \text{ kg m}^{-3}$

volume $= 12 \times 5.6 \times \frac{3.2 + 1.2}{2} = 148 \text{ m}^3$

$m = d \times v = 1 \times 10^3 \times 148 = 1.48 \times 10^5$ kg

[2]

(b) A heater raises the temperature of the water in the pool from 10 °C to 30 °C. +20°C

Calculate the energy supplied to the heater.

State an assumption you have to make.

specific thermal capacity of water = $4.2 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$

$\Delta E = mc\Delta\theta = 1.48 \times 10^5 \times 4.2 \times 10^3 \times 20$

$= 1.24 \times 10^{10}$ J

energy = ... J **[2]**

(c) Evaporation is one way in which water in a swimming pool cools down.

The Boltzmann factor can be used to model how the rate of evaporation varies with temperature.

(i) The energy required to evaporate 1.0 kg of water is 2.3×10^6 J.

Show that the energy required to remove **one molecule** of water from the pool into the air above it is about 7×10^{-20} J.

molar mass of water = 1.8×10^{-2} kg mol^{-1}
Avogadro constant = 6.0×10^{23} mol^{-1}

No of mol in 1 kg $= 1/1.8 \times 10^{-2} = 55.6$ mol

No of molecules $= 6 \times 10^{23} \times 55.6 = 3.33 \times 10^{25}$

$\therefore E = 2.3 \times 10^6 / 3.33 \times 10^{25} = 6.9 \times 10^{-20}$ J

[2]

(ii) The rate of evaporation from the pool R is estimated by

$$R = Ce^{-\frac{\varepsilon}{kT}}.$$

Explain how the Boltzmann factor $f = e^{-\frac{\varepsilon}{kT}}$ can be used to justify this equation.

Your answer should clearly link the Boltzmann factor to the behaviour of the water molecules.

Boltzmann factor $e^{-\varepsilon/kT}$ is fraction of molecules at temp T that have > energy ε the activation energy for the process. In this case for evaporation.

Molecules gain the energy to exceed ε in collisions with other molecules.

[3]

(iii) The rate of evaporation from the pool is $7.2 \times 10^{-3}\,\mathrm{kg\,s^{-1}}$ when the temperature is $+30\,°\mathrm{C}$. Estimate the rate of evaporation at a temperature of $+10\,°\mathrm{C}$.

Show your working.

$k = 1.4 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

$$R = Ce^{-\varepsilon/kT}$$

$$C = R/e^{-\varepsilon/kT} = 7.2\times10^{-3}/e^{\frac{-6.9\times10^{-2}}{1.4\times10^{-23}\times303}} = 8.34\times10^{4}$$

$$R_{10°C} = 8.34\times10^{4}\,e^{\frac{-6.9\times10^{-2}}{1.4\times10^{-23}\times283}} = 2.28\times10^{-3}\ \mathrm{kg\,s^{-1}}$$

rate of evaporation = ... $\mathrm{kg\,s^{-1}}$ [2]

[Total: 11]

Question 11 begins on page 14

11 This question is about a measurement of the half-life of protactinium.

A fresh sample of protactinium was placed in a detector of radiation.

The count rate of the sample was measured at intervals of one minute for six minutes.

The activity A, the count rate corrected for background radiation, is given in the table.

time / minutes	0	1	2	3	4	5	6
activity A / Bq	943	523	287	161	79	61	20

(a) Describe how the correction for background radiation is made.

Measure count rate with no sample and subtract from each reading in table.

[2]

(b) Use the relationships $N = N_0 e^{-\lambda t}$ and $\frac{\Delta N}{\Delta t} = -\lambda N$ to show that the results of the experiment should obey the relationship $\ln A = C - \lambda t$, where C is a constant.

$$A = \frac{-\Delta N}{\Delta t} = \lambda N$$

$$A = \lambda N = \lambda N_0 e^{-\lambda t}$$

$$\ln A = \underbrace{\ln(\lambda N_0)}_{\text{constant}} - \lambda t$$

[3]

$$\therefore \ln A = C - \lambda t$$

(c) The graph of Fig. 11.1 shows the variation of $\ln A$ with time.

$\Delta \ln A = 6.85 - 3$
$= \underline{3.85}$

$\Delta t = 6.3 \text{ min}$

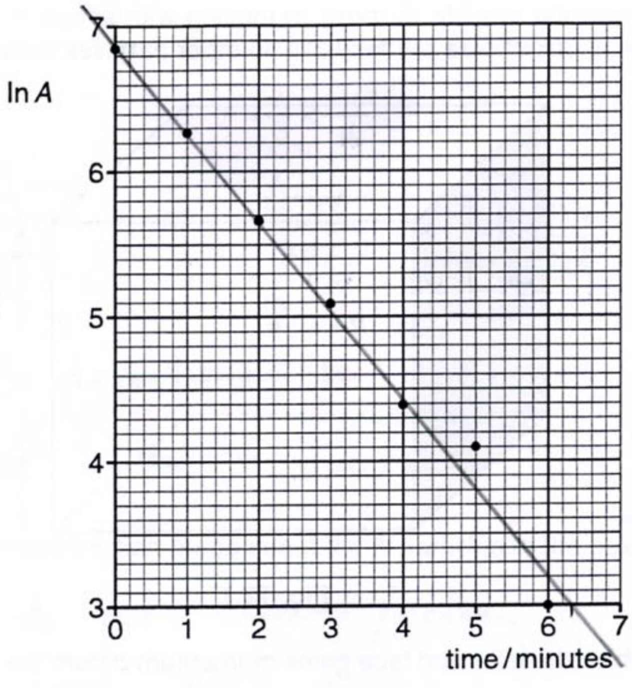


Fig. 11.1

(i) Use the graph to determine a value for the half-life of protactinium.

$\ln A = -\lambda t + C$

$\therefore$ graph has gradient of $-\lambda = \frac{-3.85}{6.3} = \lambda = 0.611 \text{ min}^{-1}$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = 1.13 \text{ min} =$

half-life = ...68.......... s **[3]**

(ii) Give a reason why some of the points on the graph of Fig. 11.1 are not expected to lie close to a straight line.

decay is a random process

[1]

[Total: 9]

12 This question is about a derivation of the relationship $PV = NkT$ for a gas from a simple model.

(a) Fig. 12.1 shows one particle of mass m moving with speed v directly towards the left-hand face of a cubical box of side d. There are no other particles in the box.

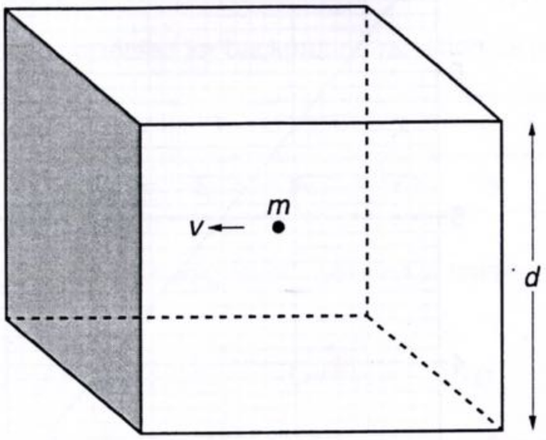


Fig. 12.1

The rate at which the left-hand face gains momentum p from the particle in the box is given by

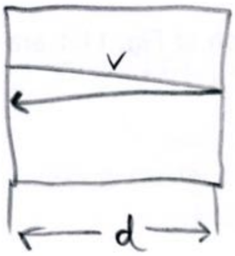
$$\frac{\Delta p}{\Delta t} = \frac{2mv}{\frac{2d}{v}}.$$

(i) State what assumption has to be made about the motion of the particle for $\Delta p = 2mv$.

on collisions with wall sign of momentum changes but magnitude does not. + Collisions are elastic

[1]

(ii) Explain why $\Delta t = \frac{2d}{v}$.



time = $\frac{\text{dist}}{\text{speed}} = \frac{2 \times d}{v}$

[1]

(b) The box now contains N particles of the gas, all with the same speed and mass, so that it models a gas.

(i) Explain why the total force F on the left-hand face of the box is given by

$$F = \frac{N}{3} \times \frac{mv^2}{d}.$$

$$\text{Force} = \frac{\Delta mv}{\Delta t} = \frac{\Delta mv}{2d/v} = \frac{\Delta mv^2}{2d}$$

There are 3 dimentions so only $\frac{1}{3}$ of 1 molecules velocity vectors are in $\longleftrightarrow$ direction $\therefore \frac{N}{3} \frac{\Delta mv^2}{2d}$. **[3]**

(ii) State another assumption made about the N particles in the box.

Particles do not interact/collide

they ignore each other. **[1]**

(c) The pressure P on the left-hand face of the box is then given by

$$P = \frac{F}{A} = \frac{Nmv^2}{3V}.$$

By making appropriate assumptions about the particles of a gas, this can be used to show that $P = \frac{NkT}{V}$.

State the assumptions required and explain how they lead to the final equation.

$$\frac{Nmv^2}{3V} = \frac{NkT}{V}$$

$$E_K = \frac{mv^2}{2} \qquad E_K = \frac{3}{2}kT$$

$$\frac{1}{2}mv^2 \propto T$$

[3]

[Total: 9]

[Section B Total: 40]

END OF QUESTION PAPER

$$\frac{Nmv^2}{3V} \xrightarrow[\text{1}]{E_K = \frac{1}{2}mv^2} \frac{N \times 2E_K}{3V} \xrightarrow[\text{2}]{E_K = \frac{3}{2}kT} \frac{N\,3kT}{3V} \rightarrow \frac{Nk}{V}$$