5A Written Answers

14.1 Gas Laws Past Questions

G494 Jan 2010

9 This question is about heated gases doing work. Fig. 9.1 shows the sequence of changes to the gas trapped in the cylinder of a petrol engine. The graph shows the changes of pressure as the gas is compressed, ignited and allowed to expand again.

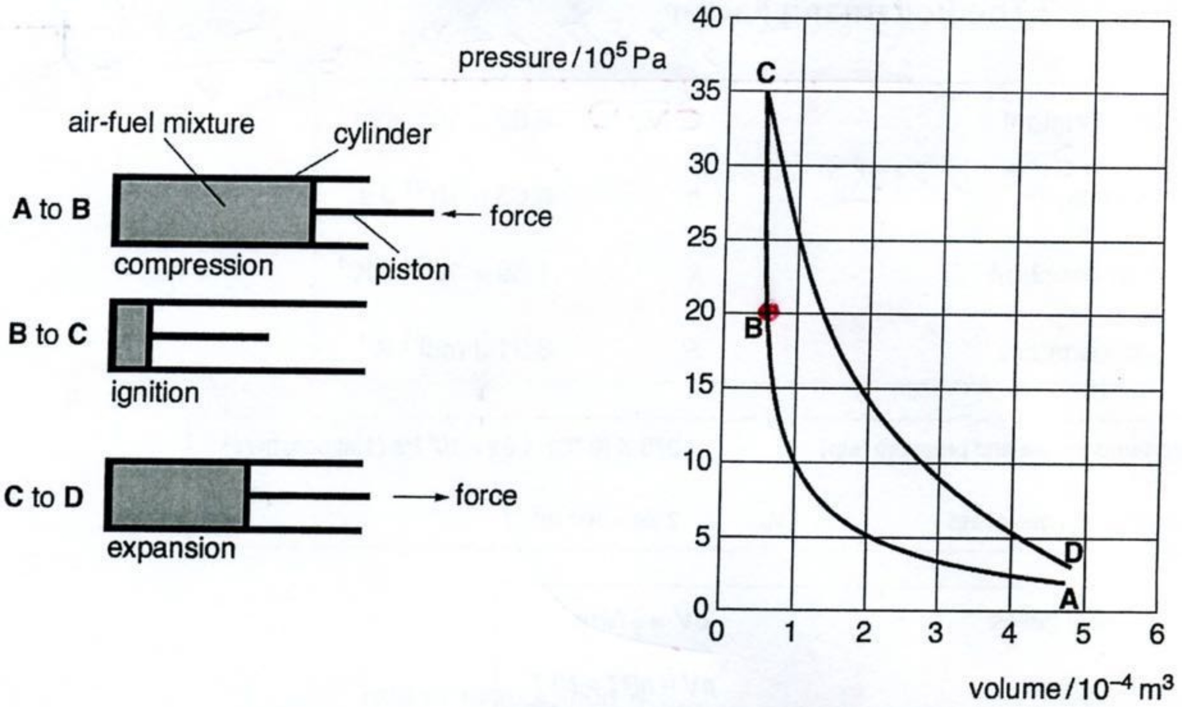


Fig. 9.1

(a) At point **A** the air-fuel mixture enters the cylinder at a pressure of 1.0×10^5 Pa and occupies a volume of 4.8×10^{-4} m^3, with a temperature of 27°C.

(i) Show that the air-fuel mixture at **A** contains about 1×10^{22} particles.
Treat the fuel-air mixture as an ideal gas.
$k = 1.4 \times 10^{-23}$ J K^{-1}

$pV = NkT$

$$N = \frac{pV}{kT} = \frac{1 \times 10^5 \times 4.8 \times 10^{-4}}{1.4 \times 10^{-23} \times 300} = \underline{1.14 \times 10^{22}}$$

[3]

(ii) It is then rapidly compressed to **B** by the piston to a volume of just 6.0×10^{-5} m^3. Using information from the graph, show that the temperature of the gas at **B** is about 500°C.

$$T = \frac{pV}{Nk} = \frac{20 \times 10^5 \times 6 \times 10^{-5}}{1.14 \times 10^{22} \times 1.4 \times 10^{-23}} = 752 \text{ K}$$

[2]

$$752 - 273 = \underline{479\,°C}$$

G494 June 2011

9 A sealed glass bottle has a volume of $7.6 \times 10^{-4}\,m^3$.

(a) The bottle contains air at a pressure of 1.2×10^5 Pa at a temperature of 15 °C.

Calculate the number of particles in the bottle.

$pV = NkT$

Boltzmann constant $k = 1.4 \times 10^{-23}\,J\,K^{-1}$

$$N = \frac{pV}{kT} = \frac{1.2\times10^5 \times 7.6\times10^{-4}}{1.4\times10^{-23}\times(15+273)} =$$

N = 2.26×10^{22} **[2]**

(b) On the axes of Fig. 9.1, sketch a graph to show how the pressure P of the gas in the bottle varies with kelvin temperature T. Assume the air behaves as an ideal gas. **[1]**

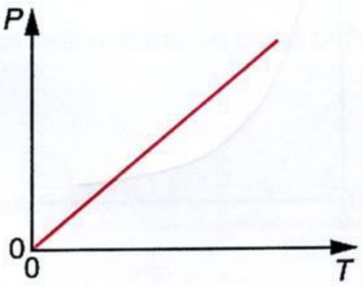


Fig. 9.1

G494 Jan 2012

9 A gas cylinder of internal volume $2.9 \times 10^{-2}\,m^3$ is filled with helium gas to a pressure of 2.1×10^7 Pa at a temperature of 290 K.
Calculate the mass of helium in the cylinder.

$R = 8.3\,mol^{-1}\,K^{-1}$
molar mass of helium is $4.0 \times 10^{-3}\,kg\,mol^{-1}$

$pV = nRT$

$$n = \frac{pV}{RT} = \frac{2.1\times10^7 \times 2.9\times10^{-2}}{8.3\times290} = 253\ \text{mol}$$

$$m = Mn = 4\times10^{-3}\times253 =$$

mass of helium = 1.01 kg **[2]**

G494 Jan 2013

7 64 mol of an ideal gas is placed in a container of volume $0.75\,m^3$ at a temperature of $36\,°C$.

(a) Calculate the pressure p of the gas in the container.

$pV = nRT$

$R = 8.3\,J\,mol^{-1}\,K^{-1}$

$$p = \frac{nRT}{V} = \frac{64 \times 8.3 \times (36 + 273)}{0.75}$$

p = 2.18×10^5 Pa **[2]**

(b) On the axes of Fig. 7.1, sketch a graph to show how the pressure of an ideal gas varies with its volume at a constant temperature.

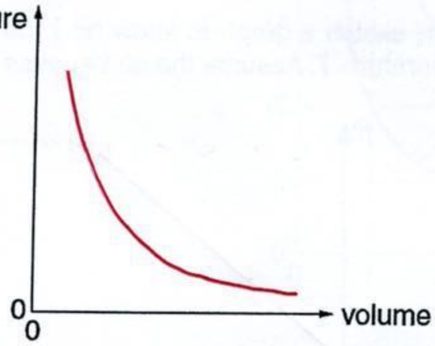
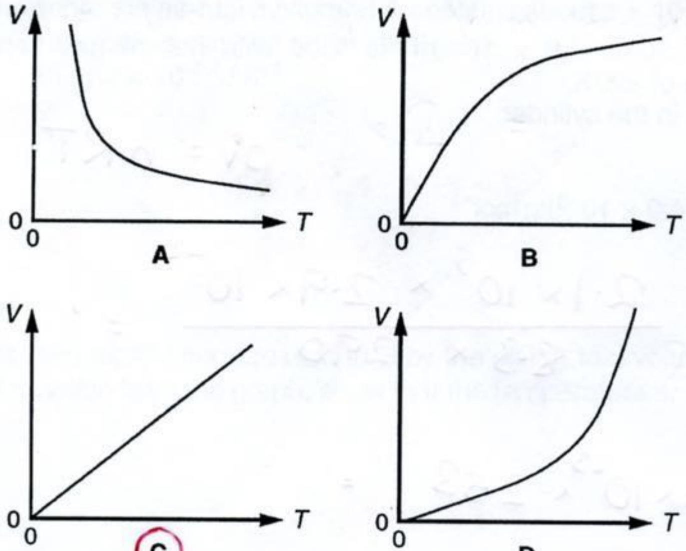


Fig. 7.1

[2]

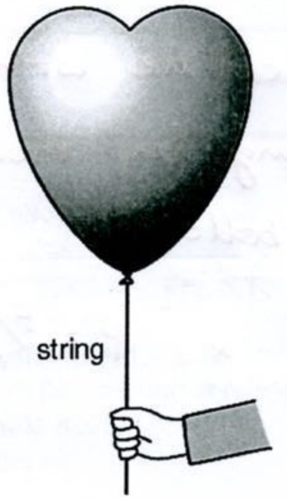
G494 June 2014

6 Which one of these four graphs (**A**, **B**, **C** or **D**) shows how the volume V of a fixed mass of gas at a constant pressure changes with kelvin temperature T?



answer C **[1]**

10 This question is about the use of helium to inflate party balloons.



(a) A balloon is fully inflated with helium gas at a temperature of 20°C (293 K) and a pressure of 1.1×10^{5} Pa. The volume of the gas in the inflated balloon is $4.5\times10^{-3}\,m^3$.

(i) Calculate the number of helium particles in the balloon.

$k = 1.4\times10^{-23}\,J\,K^{-1}$

$$N = \frac{pV}{kT} = \frac{1.1\times10^{5}\times4.5\times10^{-3}}{1.4\times10^{-23}\times293} =$$

number = 1.21×10^{23} **[3]**

(ii) Calculate the mass of helium gas in the balloon.

molar mass of helium atoms = 4.0×10^{-3} kg mol^{-1}

Avogadro constant = 6.0×10^{23} mol^{-1}

$$n = N/N_A = 1.21\times10^{23}/6\times10^{23} = 0.20\ \text{mol}$$

$$m = Mn = 4\times10^{-3}\times0.2$$

mass = 8.0×10^{-4} kg **[1]**

(b) Explain how the motion of the helium particles stops the balloon from collapsing.

Your answer should clearly link the motion of the particles to their effect on the balloon.

Helium particles collide with surface of balloon. The change in momentum results in a force on balloon.

$\Delta p / \Delta t = F$ & $p = F/A$

[3]

(c) A second identical balloon is filled with hydrogen gas.

The gas in each balloon has the same density, volume and temperature.

Calculate the pressure of the hydrogen gas. Explain your answer clearly.

molar mass of hydrogen molecules = $2.0 \times 10^{-3}\,\text{kg mol}^{-1}$

H_2 has half the mass of He

so N must be double to give same density

as $pV = NkT$ if N is double p must be double. $1.1 \times 10^5 \times 2 =$

pressure = 2.2×10^5 Pa **[3]**

G494 Jan 2012

5 Air at a pressure of 1.0×10^5 Pa and temperature 280 K is trapped in a cylinder by a piston.

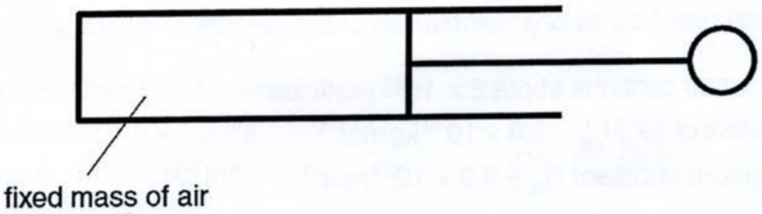


Fig. 5.1

$\frac{pV}{T} = Nk = \text{constant}$

The trapped air has an initial volume of $1.4 \times 10^{-6}\,m^3$.
The air is then compressed until its pressure is raised to 5.6×10^5 Pa.
The final temperature of the air is 320 K.
Calculate the final volume of the air.

so $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ $\therefore$ $V_2 = \frac{p_1 V_1 T_2}{T_1 p_2}$

$= \frac{1 \times 10^5 \times 1.4 \times 10^{-6} \times 320}{280 \times 5.6 \times 10^5} =$

volume = 2.86×10^{-7} m^3 **[2]**

14.2 The Kinetic Model of Gases

G494 Jan 2011

4 The air in a football has a mass of 1.1×10^{-2} kg.

(a) Show that the ball contains about 2×10^{23} particles.

molar mass of air $M_{air} = 2.9 \times 10^{-2}\,\mathrm{kg\,mol^{-1}}$

the Avogadro constant $N_A = 6.0 \times 10^{23}\,\mathrm{mol^{-1}}$

mass of air particle $= \dfrac{M_{air}}{N_A} = \dfrac{2.9 \times 10^{-2}}{6 \times 10^{23}} = 4.83 \times 10^{-26}$ kg

$N = \dfrac{\text{Mass of Air}}{\text{Mass of particle}} = 1.1 \times 10^{-2} / 4.83 \times 10^{-26} = \underline{\underline{2.28 \times 10^{23}}}$ **[1]**

(b) Calculate the mean square speed $\overline{c^2}$ of the particles in the football.

pressure of air in the ball $= 1.7 \times 10^5$ Pa

volume of air in the ball $= 5.4 \times 10^{-3}\,\mathrm{m^3}$

$pV = \frac{1}{3} Nm\overline{c^2}$

$\overline{c^2} = \dfrac{3pV}{Nm} = \dfrac{3 \times 1.7 \times 10^5 \times 5.4 \times 10^{-3}}{1.1 \times 10^{-2}}$

total mass of gas

$\overline{c^2}$ = ... 2.50×10^5 .... $\mathrm{m^2 s^{-2}}$ **[2]**

(c) The temperature of the air in the football increases during a game.

On the axes of Fig. 4.1, sketch a graph to show how the mean square speed $\overline{c^2}$ of air particles in the football varies with absolute temperature T.

Assume the air behaves as an ideal gas.

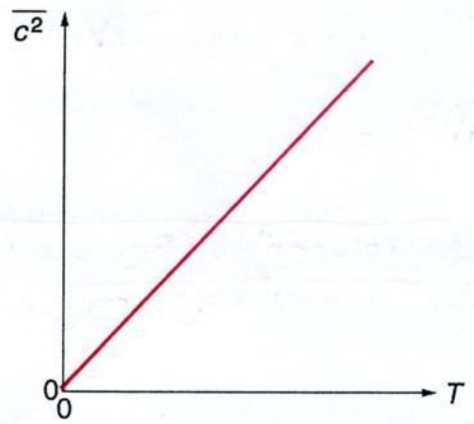


Fig. 4.1

[1]

G 494 Jan 2011

12 This question is about providing warm fresh air for people who work in large office buildings in winter.

It is recommended that the air in offices should be completely replaced once every hour.

(a) The air in an office has the following properties:

- pressure of 1.0×10^5 Pa
- volume of 1.3×10^2 m^3
- temperature of 20° C

$pV = NkT$

Show that the office contains about 3×10^{27} particles of air.

$k = 1.4 \times 10^{-23}$ J K^{-1}

$$N = \frac{pV}{kT} = \frac{1 \times 10^5 \times 1.3 \times 10^2}{1.4 \times 10^{-23} \times (273 + 20)} = \underline{3.17 \times 10^{27}}$$

[2]

(b) The air in the office is replaced every hour. On its way into the room, the cold air at 5°C is passed through a heater to raise its temperature to 20°C.

Do calculations to estimate the power of the heater.

$$\Delta E = \tfrac{3}{2} k \, \Delta T \, N = \tfrac{3}{2} \times 1.4 \times 10^{-23} \times 15 \times 3.17 \times 10^{27} = 9.99 \times 10^5 \text{ J}$$

$$P = \frac{\Delta E}{\Delta t} = \frac{9.99 \times 10^5}{60 \times 60} = \underline{277 \text{ W}}$$

power = ... W **[3]**

(c) On its way through the heater, the cold air

- increases its temperature
- has no change to its pressure

Explain how the density of the air changes as it passes through the heater.

Your answer should clearly link the change of density to the behaviour of the air particles.

$$\frac{pV}{T} = Nk = \text{constant}$$

If T increases and p is constant V must increase

Density = m/V so it must decrease.

[4]

13 Fig. 13.1 shows an experiment where liquid bromine is released into an **evacuated** tube. The brown bromine vapour is seen to fill the tube very quickly, in less than a second.

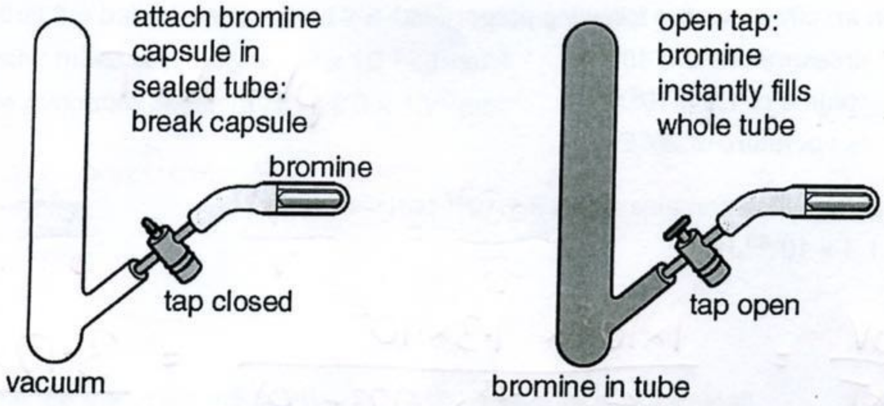


Fig. 13.1

(a) Show that the average energy of a bromine molecule at room temperature (17 °C) is about 5×10^{-21} J.

$k = 1.4 \times 10^{-23}\,\text{J K}^{-1}$

$$E_K = \frac{3}{2}kT = \frac{3}{2} \times 1.4 \times 10^{-23} \times (273 + 17)$$
$$= 6.1 \times 10^{-21}$$
(or 4.1×10^{-21} if using $E_K \approx kT$) **[2]**

(b) Calculate the mean speed of the bromine molecules in the bromine vapour.

molar mass of bromine vapour = 0.16 kg
$N_A = 6.0 \times 10^{23}\,\text{mol}^{-1}$

$$pV = \frac{1}{3}Nm\overline{c^2} = NkT \quad \therefore \quad m\overline{c^2} = 3kT$$

$$\sqrt{\overline{c^2}} = \sqrt{\frac{3kT}{m}} \qquad m = M/N_A = 0.16/6 \times 10^{23} = 2.667 \times 10^{-25}\,\text{kg}$$

$$c_{rms} = \sqrt{\overline{c^2}} = \sqrt{\frac{3 \times 1.4 \times 10^{-23} \times (273+17)}{2.667 \times 10^{-25}}} =$$

speed = 214 m s^{-1} **[3]**

(c) When liquid bromine is released into a tube full of air at atmospheric pressure, it can take up to an hour for the brown colour to completely fill the tube. This is because the bromine molecules follow a random walk through the air.

(i) Explain what is meant by a *random walk* and why the bromine molecules in the air-filled tube undergo a random walk.

Your answer should clearly link the behaviour of the molecules to their motion.

The bromine molecules collide with air molecules randomly changing the direction & magnitude of their velocities.

e.g. 

[3]

(ii) The average displacement of a particle which follows a random walk of N steps is proportional to $\sqrt{N}$. Explain how this rule can be used to justify the relationship $x = C\sqrt{t}$, where x is the average displacement of a bromine molecule in a time t and C is a constant.

The average speed is constant so

$x \propto \sqrt{N}$

The number of steps N will be proportional to the time t so $N \propto t$ and we know that $x \propto \sqrt{N}$ so $x \propto \sqrt{t}$

so $x = C\sqrt{t}$ where C is the constant of proportionality.

[3]

3 An ideal gas obeys this equation.

$$pV = \frac{1}{3} Nm\overline{c^2}$$

$\therefore \frac{Nm}{V} = \frac{3p}{\overline{c^2}}$

(a) Use the equation to show that the density D of an ideal gas is given by the relationship:

$$D = \frac{3p}{c^2}$$

Nm = mass of gas.

$D = \frac{\text{mass}}{\text{volume}} = \frac{Nm}{V} = \frac{3p}{\overline{c^2}}$

[1]

(b) Use this information to calculate the typical speed of a gas particle in air.

$p = 1.0 \times 10^5$ Pa
$D = 1.2\,\text{kg m}^{-3}$

$\sqrt{\overline{c^2}} = \sqrt{3p/D} = \sqrt{\frac{3 \times 1 \times 10^5}{1.2}} =$

speed = ...500........ m s^{-1} **[1]**

(c) A gas particle takes a few minutes to travel about a metre through air.

Explain why it takes so long to cover this distance.

Gas particles change direction randomly as they collide with other gas molecules.

[2]

12 This question is about deriving the equation $PV = \frac{Nm}{3}\overline{c^2}$ for an ideal gas.

Although the usual model considers particles in a rectangular box, a student chooses to model an ideal gas as follows:

- the gas is in a spherical container of radius r
- each particle has mass m and speed v
- all particles move at right angles to the walls of the container

(a) Fig. 12.1 shows a single particle approaching the container wall.

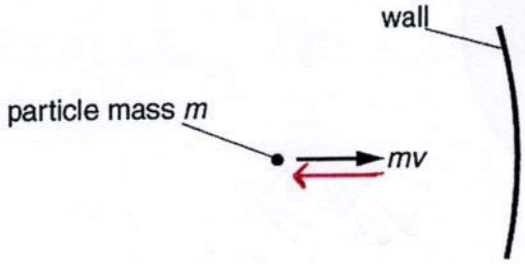


Fig. 12.1

(i) The arrow on Fig. 12.1 represents the momentum of the particle before it hits the wall. Draw another arrow to represent the momentum of the particle **after** it has hit the wall. **[1]**

(ii) Explain why the change of momentum Δp of the wall when the particle bounces off it is given by $\Delta p = 2mv$, towards the right.

$\Delta p = mv - -mv = 2mv$

[2]

(iii) Explain why the number of times n that the particle bounces off the same point on the wall in each second is given by $n = \frac{v}{4r}$.

The radius of the sphere is r.

Distance to get back to wall is 4r

$\text{time} = \frac{\text{dist}}{\text{vel}} = \frac{4r}{v}$

No of collisions in 1s $= {}^{1s}\!/\frac{4r}{v} = \frac{v}{4r}$

[1]

(b) The force F on the wall due to a single particle is given by

$$F = n\Delta p = \frac{mv^2}{2r}.$$

The student then considers the effect of all N particles in the container, so that bounces are evenly spread over a hemisphere, as shown in Fig. 12.2.

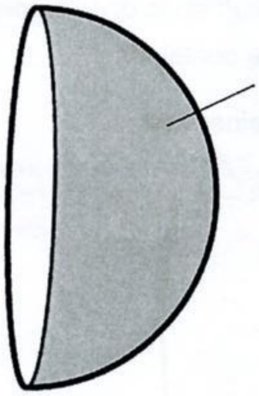


Fig. 12.2

(i) Show that the pressure P on the wall of the container is given by

$$P = \frac{Nm}{3V} v^2$$

where $V = \frac{4}{3}\pi r^3$ is the volume of the spherical container.

$F = \frac{mv^2}{2r}$ & for N molecules $F = \frac{Nmv^2}{2r}$

$P = \frac{F}{A} = \frac{Nmv^2}{2r\ 2\pi r} = \frac{Nm}{4\pi r^3} v^2 = \frac{Nm}{3V} v^2$

$(4\pi r^3 = 3V)$

[3]

(ii) Although the student's model produces the correct relationship, many aspects of it are unrealistic. Describe three assumptions of this model that are unrealistic.

Your answer should have correct spelling, punctuation and grammar.

particles don't collide with each other

collisions are perpendicular to surface

collisions are elastic (conserve energy)

[3]

$pV = NkT = \frac{1}{3}Nm\bar{c}^2$

G494 June 2014

4 The atmosphere of Mars is mostly carbon dioxide at a mean temperature of –63 °C.

Estimate the speed v of carbon dioxide molecules at this temperature.

mass of a carbon dioxide molecule = 7.3×10^{-26} kg

$k = 1.4 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

$$\sqrt{\bar{c^2}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.4 \times 10^{-23} \times (273-63)}{7.3 \times 10^{-26}}}$$

$$\left(\begin{array}{l} \text{or } \frac{1}{2}mv^2 = kT \\ v = 284\,\mathrm{ms^{-1}} \end{array}\right)$$

v = ...348...... m s^{-1} [3]

G494 June 2014

7 This question is about the random walk of a molecule through air.

(a) Explain why the molecule does a random walk.

Collisions with other molecules randomly change the direction & magnitude of their velocities

[2]

(b) A typical molecule in still air is displaced by a distance of 5 mm in a time of 1 s.

Explain why it will take 100 s for the molecule to be displaced by a distance of 50 mm.

displacement $\propto \sqrt{N}$ & $N \propto t$ so disp $\propto \sqrt{t}$

$\sqrt{100} = 10$ and $5\,\mathrm{mm} \times 10 = 50\,\mathrm{mm}$

[2]

12 This question is about a derivation of the relationship $PV = NkT$ for a gas from a simple model.

(a) Fig. 12.1 shows one particle of mass m moving with speed v directly towards the left-hand face of a cubical box of side d. There are no other particles in the box.

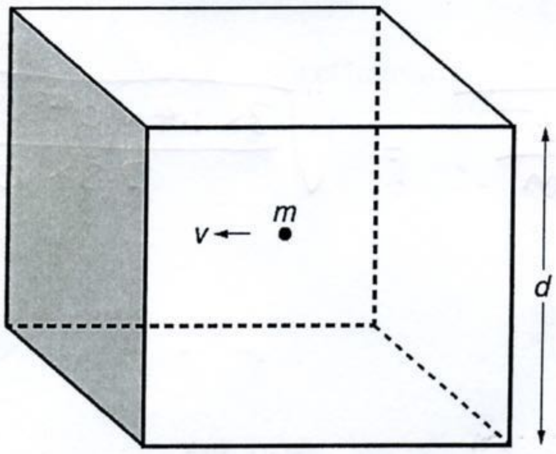


Fig. 12.1

The rate at which the left-hand face gains momentum p from the particle in the box is given by

$$\frac{\Delta p}{\Delta t} = \frac{2mv}{\frac{2d}{v}}.$$

(i) State what assumption has to be made about the motion of the particle for $\Delta p = 2mv$.

Collision results in equal & opposite momentum (or velocity)

[1]

(ii) Explain why $\Delta t = \frac{2d}{v}$.

$\Delta t = \frac{2d}{v}$ as time $= \frac{\text{dist}}{\text{vel}}$ & particle must travel back and forth $\underset{d}{\overset{d}{\rightleftarrows}}$ between collisions

[1]

$d + d = 2d$.

(b) The box now contains N particles of the gas, all with the same speed and mass, so that it models a gas.

(i) Explain why the total force F on the left-hand face of the box is given by

$$F = \frac{N}{3} \times \frac{mv^2}{d}.$$

There are N particles and their forces add together
There are 3 dimentions so only $\frac{1}{3}$ of the velocity vectors will be in any one dimention.

[3]

(ii) State another assumption made about the N particles in the box.

They all have same velocity, v
or They do not collide with each other

[1]

(c) The pressure P on the left-hand face of the box is then given by

$$P = \frac{F}{A} = \frac{Nmv^2}{3V}.$$

By making appropriate assumptions about the particles of a gas, this can be used to show that $P = \frac{NkT}{V}$.

State the assumptions required and explain how they lead to the final equation.

Average E_K of particles $= \frac{mv^2}{2}$

$\frac{mv^2}{2} = \frac{3}{2} kT \quad \therefore \quad mv^2 = 3kT$

$p = \frac{Nmv^2}{3V} = \frac{N3kT}{3V} = \frac{NkT}{V}$

[3]

14.3 Energy In Matter Past Questions

G494 Jan 2012

8 Slabs of aluminium with insulated handles are used in some restaurants to keep dishes hot at the table.

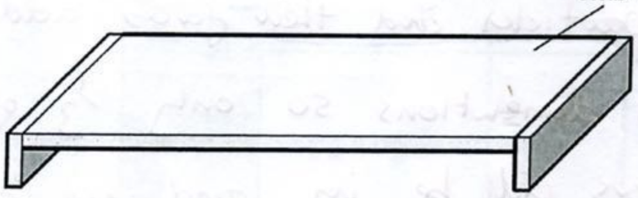


Fig. 8.1

(a) Show that an aluminium slab with dimensions 0.40 m × 0.20 m × 0.01 m will have a mass of about 2 kg.

density of aluminium = 2700 kg m^{-3}

m = density × volume
$= 2700 \times 0.4 \times 0.2 \times 0.01$
$= 2.16$ kg

[1]

(b) Calculate the thermal energy released by the plate as it cools from an initial temperature of 100 °C to room temperature of 20 °C.

specific thermal capacity of aluminium = 920 J kg^{-1} °C^{-1}

$\Delta E = mc\Delta\theta$

$= 2.16 \times 920 \times 80$

$= 1.6 \times 10^5$ J

energy released = ... J **[2]**

8 A student uses the apparatus shown in Fig. 8.1 to measure the specific thermal capacity *c* of a block of copper.

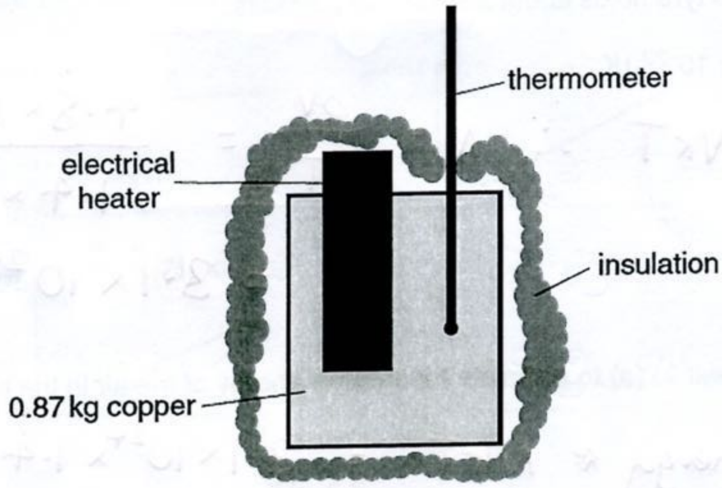


Fig. 8.1

The temperature of the block of mass 0.87 kg rises from room temperature 294 K to 308 K when 5000 J is transferred from the electrical heater.

(a) Calculate a value for the specific thermal capacity *c* of the block. $\Delta E = mc\Delta\theta$

$$c = \frac{\Delta E}{m\Delta\theta} = \frac{5000}{0.87 \times (308-294)} = \underline{4.1 \times 10^2}$$

$c =$ ....4.1×10^2.... $\mathrm{J\,kg^{-1}\,K^{-1}}$ **[1]**

(b) Suggest why the presence of the heater and thermometer means that your answer to **(a)** is only an approximation.

Some of the 5000J transferred from the electrical heater will be used to heat up the heater itself and the thermometer. **[1]**

G494 June 2010

6 The air in a car tyre has a volume of 2.5×10^{-2} m^3 and a pressure of 4.8×10^5 Pa when it has a temperature of 280 K.

(a) Show that the tyre holds about 3×10^{24} air particles.

$k = 1.4 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

$$\text{In } pV = NkT \quad \therefore \quad N = \frac{pV}{kT} = \frac{4.8\times10^{5} \times 2.5\times10^{-2}}{1.4\times10^{-23} \times 280}$$

$$= 3.1\times10^{24}$$

[1]

(b) Use your answer to **(a)** to estimate the internal energy of the air in the tyre.

Internal Energy $\approx NkT = 3.1\times10^{24} \times 1.4\times10^{-23} \times 280$

internal energy = 1.2×10^{4} J **[1]**

The Boltzmann Factor Past Questions 15.1 and 15.2

G494 Jan 2011

6 Which one of the graphs of Fig. 6.1 best shows how the Boltzmann factor $e^{-E/kT}$ varies with absolute temperature T?

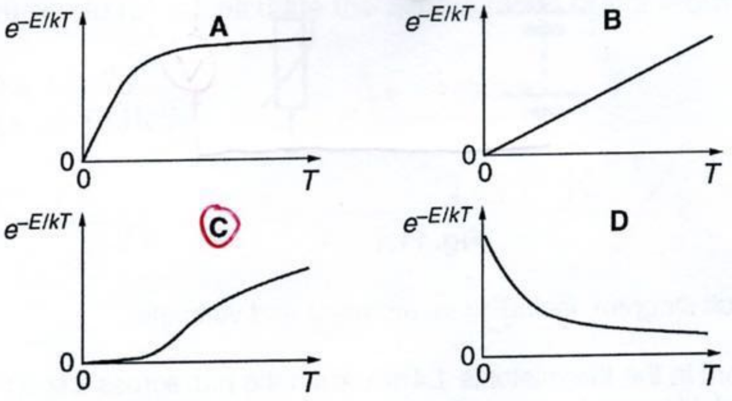


Fig. 6.1

answer C [1]

7 The energy ε required to change the state of water from liquid to gas is 6.9×10^{-20} J per particle.

(a) Calculate the ratio $\frac{\varepsilon}{kT}$ for a single water particle at $T = 300$ K.

$k = 1.4 \times 10^{-23}$ J K^{-1}

$$= \frac{6.9 \times 10^{-20}}{1.4 \times 10^{-23} \times 300}$$

$\frac{\varepsilon}{kT} =$ 16.4 [1]

(b) Calculate the value for the Boltzmann factor of a water particle at 300 K.

$$e^{-\varepsilon/kT} = e^{-16.4} =$$

Boltzmann factor = 7.33×10^{-8} [1]

(c) Although the Boltzmann factor is very small, a puddle of water at 300 K evaporates over a space of a few hours.

Two of the statements below, when taken together, provide an explanation for this.

- T The rate at which particles in the liquid collide with each other is very large. ✓
- T Only particles close to the surface of the liquid can escape. ×
- T Each particle collides with others in the liquid many times. ×
- F The temperature of the liquid rises as particles escape from it. ×
- T The energy of a particle can change each time it collides with others. ✓

Put a tick (✓) in the box next to the each of the **two** statements required. [1]

G494 Jan 2012

11 Fig. 11.1 is an incomplete circuit diagram to measure the conductance of an electrical component called a thermistor.

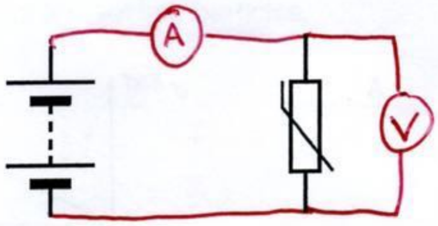


Fig. 11.1

(a) Complete the circuit diagram, including an ammeter and voltmeter. **[2]**

(b) At 300 K, the current in the thermistor is 1.4 mA when the p.d. across it is 5.6 V. Show that the conductance of the thermistor is about 3×10^{-4} S.

$$G = I/V = \frac{1.4 \times 10^{-3}}{5.6} = 2.5 \times 10^{-4} \text{ S}$$

[1]

(c) The electrical behaviour of a thermistor can be modelled as follows:

- most electrons are bound to atoms
- those few electrons with an extra energy $\mathcal{E}$ are able to move freely

(i) Use ideas about the Boltzmann factor to explain why the conductance of a thermistor increases with increasing temperature.

Your answer should use correct spelling and grammar.

The BF gives the fraction of electrons with energy $\mathcal{E}$ and hence the fraction able move freely. At a higher temperature BF increases - the electrons are more likely to gain enough energy during random exchanges to move freely

[3]

(ii) The Boltzmann factor can be used with the model to predict that the conductance G of the thermistor at temperature T is given by the relationship

$$G = G_0 e^{\frac{-\varepsilon}{kT}}.$$

Use your answer to **(b)** to calculate the conductance of the thermistor at 400 K.

$\varepsilon = 5.0 \times 10^{-20}$ J
$k = 1.4 \times 10^{-23}$ J K^{-1}

At 300K

$$G_0 = G / e^{-\varepsilon/kT} = 2.5 \times 10^{-4} / e^{(-5\times10^{-20}/1.4\times10^{-23}\times 300)}$$

$$= 37.0\,\text{S}$$

So at 400K

$$G = 37.0 \times e^{(-5\times10^{-20}/1.4\times10^{-23}\times 400)}$$

$$=$$

conductance = 4.9×10^{-3} S **[3]**

G494 Jan 2013

11 This question is about the internal energy of steam in a power station.

(a) Steam is created in a power station by heating water in the boiler. This is effectively a box containing only liquid water and steam, as shown in Fig. 11.1.

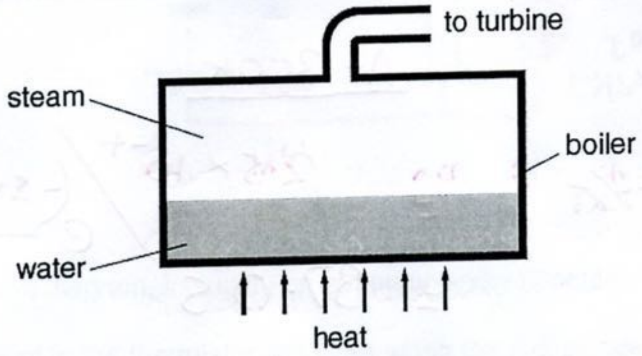


Fig. 11.1

(i) Explain why the number of molecules N in the steam at temperature T is approximated by

$$N \propto e^{-\varepsilon/kT}$$

where ε is the energy required to move a molecule from the water to the steam.

$e^{-\varepsilon/kT}$ is the Boltzmann factor that gives the fraction of particles with energy greater than ε. In this case ε is the energy for a particle to be steam.

[2]

(ii) Show that the pressure p of the steam in the boiler at temperature T is given by

$$p = CTe^{-\varepsilon/kT}$$

where C is a constant. Assume that steam in the boiler behaves like an ideal gas with a constant volume V.

$pV = NkT \quad \therefore \quad p = \frac{kTN}{V} \propto \frac{kTe^{-\varepsilon/kT}}{V}$

k & V are constant so

$p = CTe^{-\varepsilon/kT}$

[2]

(b) Describe and explain how the energy of a water molecule in steam at a constant temperature changes with time.

Your answer should clearly explain the changes of energy of the water molecule.

The frequent collisions with other particles transfers energy randomly so the energy changes randomly. The average energy will not change though.

[3]

(c) Assuming that steam behaves as an ideal gas, its **energy density**, the internal energy E_{int} per unit volume V, is given by

$$\frac{E_{int}}{V} = 3p.$$

(i) Calculate the energy density of the steam as it leaves the boiler at a pressure of 2.2×10^7 Pa.

For unit volume $V = 1\ m^3$

$\therefore\ E_{int} = 3p = 3 \times 2.2 \times 10^7$ Pa

energy density = 6.6×10^7 Jm^{-3} **[1]**

(ii) Using **(a)(ii)** sketch a graph on the axes of Fig. 11.2 to show how the energy density of steam in the boiler varies with temperature.

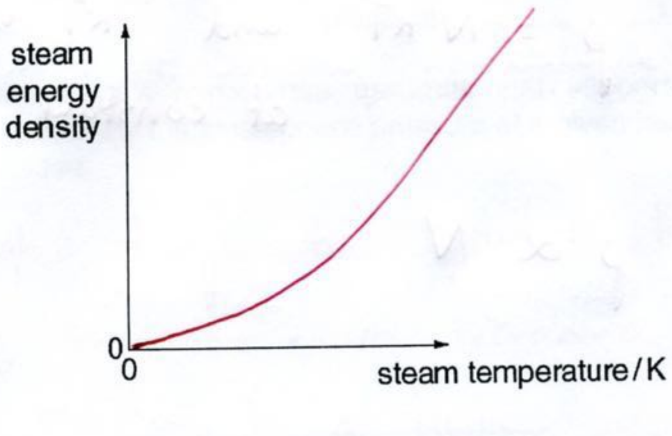


Fig. 11.2

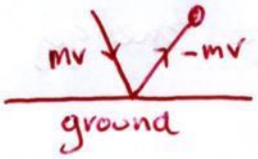
[1]

10 This question is about how atmospheric pressure changes with height above the ground.

(a) Explain how the gas particles in the Earth's atmosphere are able to exert a downwards force on the ground.

Your answer should clearly link the force on the ground to the behaviour of the gas particles.

You should use appropriate technical terms in your answer.



Particles bounce of ground so have change in momentum. This requires a force giving an equal and opposite force on the ground.

Force $= \Delta p / \Delta t$

[3]

(b) A simple model of the Earth's atmosphere assumes that all parts of it have the same temperature *T*.

(i) Use the ideal gas equation $pV = NkT$ to show that the pressure of a gas at constant temperature is proportional to the number of particles per unit volume.

$pV = NkT \quad \therefore \quad p = \frac{NkT}{V}$

For unit volume $V = 1\,m^3$

[1]

so $p = NkT$ and kT is constant at constant T

$\therefore \quad p \propto N$

(ii) By considering the energy required to move a gas particle of mass m to a height h above the ground, use the Boltzmann factor to justify the expression

$$p = p_0 e^{-\frac{mgh}{kT}}.$$

p is the pressure at height h
p_0 is the pressure at ground level
g is the gravitational field strength
T is the temperature of the air

Fraction of particles at height h = $e^{-\varepsilon/kT}$
where ε is the energy of particle at height h. $mgh = \varepsilon$

[2]

(iii) Atmospheric pressure is 100 kPa at ground level when the temperature is 290 K. Calculate the pressure at a height of 2.0 km, assuming the temperature remains constant.

$m = 4.9 \times 10^{-26}$ kg
$k = 1.4 \times 10^{-23}$ J K^{-1}
$g = 9.8$ N kg^{-1}

$$p = 100\,\text{kPa} \times e^{\left(\frac{-4.9\times10^{-26}\times 9.8\times 2\times10^{3}}{1.4\times10^{-23}\times 290}\right)}$$
$$= 78.9\,\text{kPa}$$

pressure = 7.9×10^{4} Pa **[1]**

(c) The model of the Earth's atmosphere assumes that all parts of it have the same temperature T. Explain why the atmospheric pressure at a given height h increases when the temperature T is raised.

At higher temperature particles have more energy so more can reach a particular height resulting in a higher pressure.

[2]

$p \propto N$

13 This question is about using the Boltzmann factor to explain a property of liquid ethanol. The fluidity ϕ of ethanol is a measure of how easily it flows.
The table shows how the fluidity of ethanol varies with temperature.

temperature / K	fluidity / $N^{-1}m^2s^{-1}$
273	530
298	920
323	1500
348	2200

+ ↓25 ↓25 ↓25

1.74
1.63
1.47

Not constant ratio so not exponential.

(a) Perform a test on the data in the table to show that the fluidity does **not** increase exponentially with temperature. Show your method clearly.

See above

[2]

(b) Liquid ethanol can be modelled as particles very close to one another, with the liquid only able to change its shape when particles break free from their neighbours and move to a different place. This allows the variation of fluidity ϕ with temperature T to be modelled with the Boltzmann factor, leading to the relationship

$$\phi = \phi_0 e^{-\frac{\varepsilon}{kT}}.$$

(i) In this model, what is the quantity ε in the relationship $\phi = \phi_0 e^{-\frac{\varepsilon}{kT}}$?

energy required for particle to move / break free

[1]

(ii) In this model, what is the quantity kT in the relationship $\phi = \phi_0 e^{-\frac{\varepsilon}{kT}}$?

average energy of particles

[1]

(iii) On the axes of Fig. 13.1, sketch a graph for $\phi = \phi_0 e^{-\frac{\varepsilon}{kT}}$.

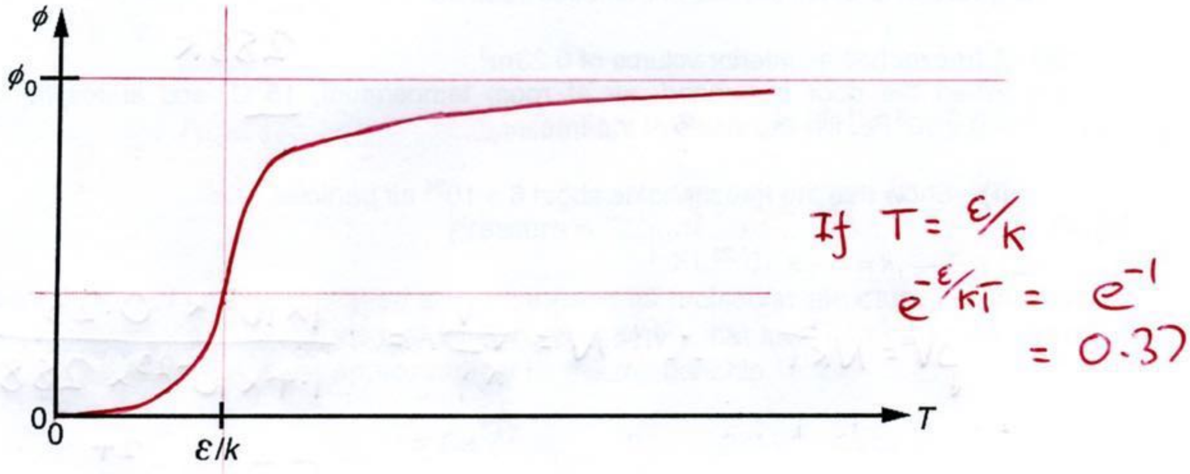


Fig. 13.1

[3]

(c) Explain how the Boltzmann factor accounts for the way that the fluidity of ethanol increases with temperature.

The BF, $e^{-\varepsilon/kT}$ increases with increasing temperature so fraction of particles able to move increases.

[2]

11 This question is about the use of domestic freezers.

(a) A freezer has an interior volume of $0.23\,m^3$.

288K

When the door is opened, air at room temperature, 15 °C, and atmospheric pressure, 1.0×10^5 Pa, fills the inside of the freezer.

(i) Show that the freezer holds about 6×10^{24} air particles.

$k = 1.4 \times 10^{-23}\,J\,K^{-1}$

$$pV = NkT \quad \therefore \quad N = \frac{pV}{kT} = \frac{1\times10^5 \times 0.23}{1.4\times10^{-23}\times 288}$$

$$= 5.7\times10^{24}$$

[2]

(ii) When the door is shut the air is cooled to –20 °C. $\Delta T = 35\,K$

Estimate the total energy removed from the air particles to make this happen.

$$\Delta E = \tfrac{3}{2}Nk\Delta T = \tfrac{3}{2}\times 5.7\times10^{24}\times 1.4\times10^{-23}\times 35$$

can leave out $\frac{3}{2}kT \approx kT$

energy = 4.2×10^3 J **[2]**

(b) The pressure of the air inside the freezer drops as it cools to –20 °C.

(i) Explain why the pressure drops in terms of the behaviour of the particles.

Your answer should clearly link the change in pressure to the behaviour of the particles.

At a lower temperature the particles have less kinetic energy so move slower. At lower velocity change in momentum due to collisions is less and collisions are less frequent. The average force is less and hence the pressure is lower

[3]

(ii) Calculate the final pressure inside the freezer assuming that it is sealed when the door is shut.

$pV = NkT$

$p = \frac{NkT}{V} = 5.7\times 10^{24} \times 1.4\times 10^{-23} \times 253 / 0.23$

pressure = 8.8×10^{4} Pa **[2]**

(c) The spoiling of food can be modelled as a change in its molecular structure which requires an activation energy ε. This means that the length of time τ that food can be safely stored at absolute temperature T is given approximately by the relationship

$$\tau = Ce^{+\frac{\varepsilon}{kT}}$$

where C is a constant.

If food which spoils after 2 days at 15 °C can be made to last for 500 days in a freezer at –20 °C, do a calculation to estimate a value for ε.

$k = 1.4 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

$$\frac{2}{500} = \frac{\tau_{15}}{\tau_{-20}} = \frac{e^{\varepsilon/k\times 288}}{e^{\varepsilon/k\times 253}} = e^{\frac{\varepsilon}{k}\left(\frac{1}{288} - \frac{1}{253}\right)}$$

$$4\times 10^{-3} = e^{\varepsilon \times -3.4\times 10^{19}}$$

$$\ln 4\times 10^{-3} = -3.4\times 10^{19}\,\varepsilon$$

$$\varepsilon = \frac{\ln 4\times 10^{-3}}{-3.4\times 10^{19}} =$$

ε = 1.6×10^{-19} J **[3]**

13 This question is about an ion thruster, a type of rocket for accelerating spacecraft.

The main details of an ion thruster are shown in Fig. 13.1.

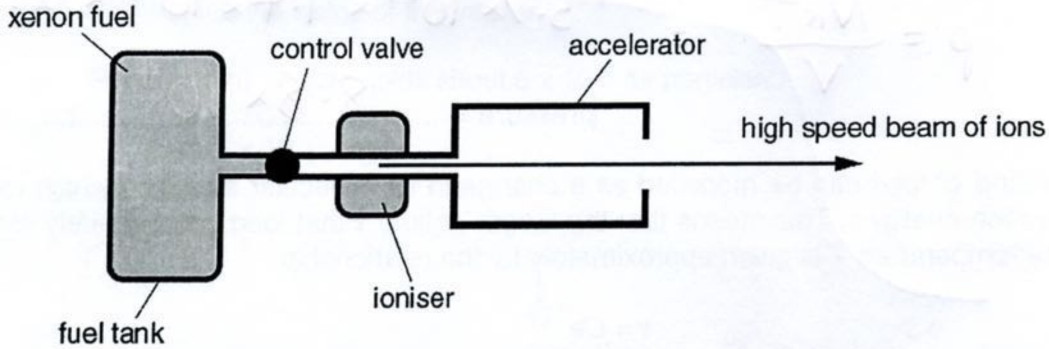


Fig. 13.1

The thruster operates as follows:

- xenon gas in the fuel tank passes into the ioniser
- the ioniser removes an electron from each xenon atom to make an ion
- the accelerator transfers kinetic energy to each ion
- the ions leave the thruster at high speed

(a) The ions enter the accelerator with negligible kinetic energy.

Each ion leaves with a kinetic energy of 4.0×10^{-16} J.

(i) Show that the momentum transferred to each ion by the accelerator is about $1 \times 10^{-20}\,\mathrm{kg\,m\,s^{-1}}$.

mass of xenon ion = 2.2×10^{-25} kg

$$E_K = \frac{mv^2}{2} \quad \therefore \quad v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2 \times 4 \times 10^{-16}}{2.2 \times 10^{-25}}}$$

$$= 6.03 \times 10^4\ \mathrm{ms^{-1}}$$

$$p = mv = 2.2 \times 10^{-25} \times 6.03 \times 10^4 = \underline{1.33 \times 10^{-20}\ \mathrm{kgms^{-2}}}$$

[3]

(ii) 3.6×10^{17} ions leave the accelerator every second.

Calculate the force on the ion thruster.

$$F = \frac{\Delta p}{\Delta t} = \frac{3.6 \times 10^{17} \times 1.33 \times 10^{-20}}{1} =$$

force = 4.78×10^{-3} N **[1]**

(iii) The combined mass of the ion thruster and spacecraft is 860 kg.

Calculate the change in speed of the spacecraft when the thruster operates for a year. Neglect any change of mass of the spacecraft.

1 year = 3.2×10^7 s

$a = F/m = \dfrac{4.78 \times 10^{-3}}{860} = 5.55 \times 10^{-6}\ \mathrm{ms^{-2}}$

$v - u = at = 5.55 \times 10^{-6} \times 3.2 \times 10^{7} =$

change of speed = ...178...... m s^{-1} **[2]**

(b) It has been suggested that the ioniser could simply be a tube heated to a high temperature.

(i) Explain why some of the gas in the tube would be ionised.

Your answer should clearly link the ionisation of the gas to its temperature.

At a high temperature the BF for the ionization energy will be large enough. A few molecules will randomly gain enough energy to be ionized.

[3]

(ii) The tube is heated to a temperature of 1400 K. The energy ε required to remove an electron from a xenon atom in the ioniser is 2.0×10^{-18} J. By calculating the value of the Boltzmann factor $f = e^{\frac{-\varepsilon}{kT}}$ for the gas in the tube at 1400 K comment on the feasibility of the suggested ioniser.

$k = 1.4 \times 10^{-23}$ J K^{-1}

$f = e^{-2 \times 10^{-18} / 1.4 \times 10^{-23} \times 1400} = 4.8 \times 10^{-45}$

this is a very small proportion - the rate of ion generation will be small.

$f =$ .... **[2]**

10 This question is about heating the water in a swimming pool which has the cross-sectional shape shown in Fig. 10.1.

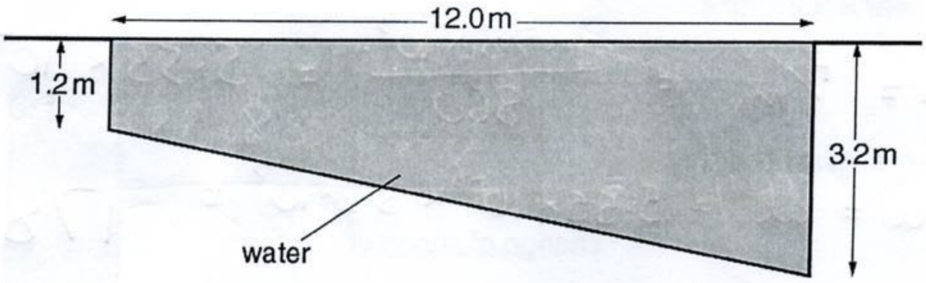


Fig. 10.1

(a) The pool has a constant width of 5.6 m.

Show that it contains about 1.5×10^5 kg of water.

density of water = $1.0 \times 10^3 \text{kg m}^{-3}$

$$\text{Volume} = \frac{3.2 + 1.2}{2} \times 12 \times 5.6 = 148 \text{ m}^3$$

$$\text{mass} = \text{density} \times \text{volume} = 1 \times 10^3 \times 148 = \underline{1.48 \times 10^5 \text{ kg}}$$

[2]

$\Delta T = 20\text{K}$

(b) A heater raises the temperature of the water in the pool from 10 °C to 30 °C.

Calculate the energy supplied to the heater.

State an assumption you have to make. → No other energy transfers to/from the water

specific thermal capacity of water = $4.2 \times 10^3 \text{J kg}^{-1}\text{K}^{-1}$

$$\Delta E = mc\Delta\theta = 1.48 \times 10^5 \times 4.2 \times 10^3 \times 20 =$$

energy = 1.24×10^{10} J **[2]**

(c) Evaporation is one way in which water in a swimming pool cools down.

The Boltzmann factor can be used to model how the rate of evaporation varies with temperature.

(i) The energy required to evaporate 1.0 kg of water is 2.3×10^6 J.

Show that the energy required to remove **one molecule** of water from the pool into the air above it is about 7×10^{-20} J.

molar mass of water = 1.8×10^{-2} kg mol^{-1}
Avogadro constant = 6.0×10^{23} mol^{-1}

No of molecules in 1 kg $= \frac{1.0 \text{ kg}}{1.8 \times 10^{-2}} \times 6 \times 10^{23} = 3.33 \times 10^{25}$

Energy per molecule $= 2.3 \times 10^{6} / 3.33 \times 10^{25}$

$= 6.9 \times 10^{-20}$ J

[2]

(ii) The rate of evaporation from the pool R is estimated by

$$R = Ce^{-\frac{\varepsilon}{kT}}.$$

Explain how the Boltzmann factor $f = e^{-\frac{\varepsilon}{kT}}$ can be used to justify this equation.

Your answer should clearly link the Boltzmann factor to the behaviour of the water molecules.

BF gives the fraction of molecules with enough energy to evaporate through random collisions

[3]

303 K

(iii) The rate of evaporation from the pool is $7.2 \times 10^{-3}\,kg\,s^{-1}$ when the temperature is +30 °C. Estimate the rate of evaporation at a temperature of +10 °C. 283K

Show your working.

$$C = \frac{e^{-E/kT}}{R}$$

At 30°C

$k = 1.4 \times 10^{-23}\,J\,K^{-1}$

$$C = \frac{e^{\left(-6.9\times10^{-20}/1.4\times10^{-23}\times303\right)}}{7.2\times10^{-3}} = 8.34\times10^{4}$$

At 10°C

$$R = 8.34\times10^{4}\,e^{\left(-6.9\times10^{-20}/1.4\times10^{-23}\times283\right)} =$$

rate of evaporation = 2.28×10^{-3} $kg\,s^{-1}$ **[2]**