t test : Out into space 1

9 This question is about the physics of hitting a golf ball.
Fig. 9 shows how the force exerted by a golf club on a ball varies during time of contact.

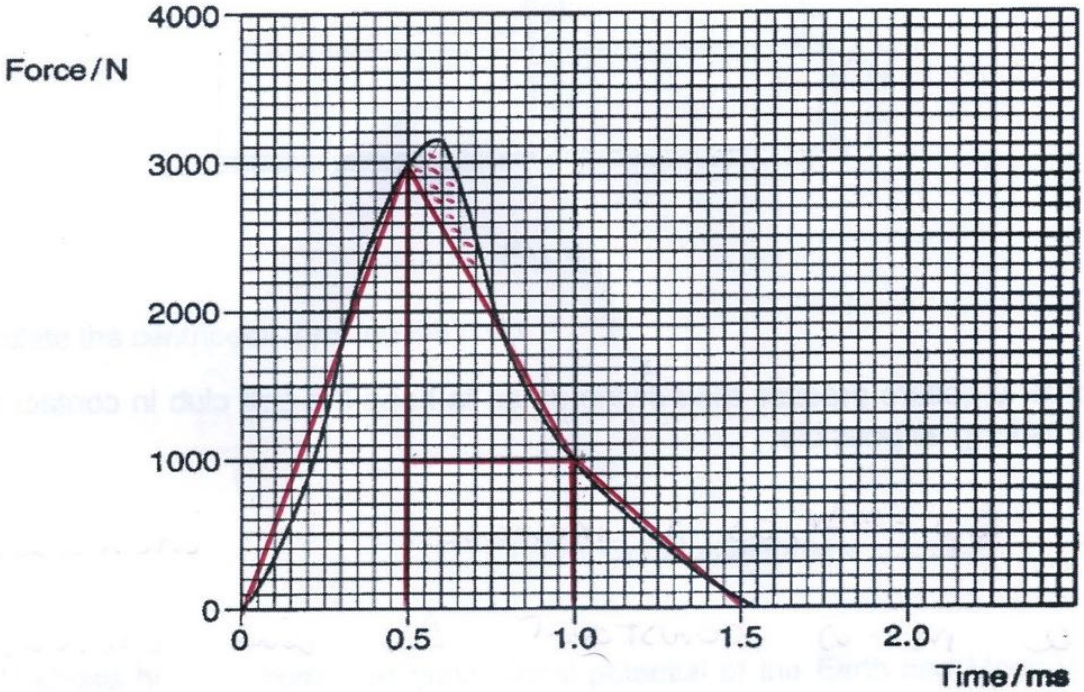


Fig. 9

(a) Show that the change of momentum of the ball during the time of contact is approximately $2\,kg\,m\,s^{-1}$.

$Ft = \Delta p = \text{area} = \frac{1}{2} \times 0.5 \times 3000 = 750$

$+ \frac{1}{2} \times 0.5 \times 2000 = 500$

$+ 0.5 \times 1000 = 500$

$+ \frac{1}{2} \times 0.5 \times 1000 = 250$

$+ 11 \times 0.05 \times 100 = 55$

$= 2055\ mNs = 2.1\ Ns$

$= \underline{2.1\ kg\,m\,s^{-1}}$

[3]

(b) The mass of the golf ball is 0.045 kg.

Calculate the speed of the golf ball at the instant it leaves the club.

$$\Delta v = \frac{\Delta p}{m} = \frac{2\ \text{kgms}^{-1}}{0.045\ \text{kg}} = \underline{\underline{44\ \text{ms}^{-1}}}$$

[3]

(c) Explain why golfers practise their swing so as to keep the golf club in contact with the ball for as long as possible.

$Ft = \Delta p = m\Delta v$, as t increases Δp increases and since m is constant Δv will increase and ball will travel forther.

[3]

4 Fig. 4 shows a child sitting on a playground roundabout. She is moving at a speed of $1.9\,m\,s^{-1}$. Her mass is 25 kg.

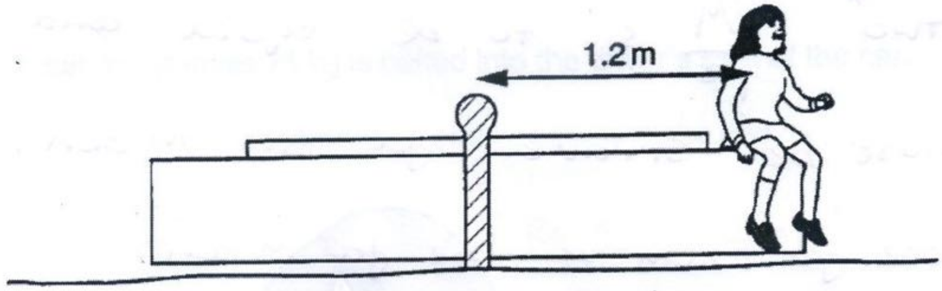


Fig. 4

Calculate the centripetal force on the child.

$F = \frac{mv^2}{r} = 25\,kg \times (1.9\,ms^{-1})^2 / 1.2\,m$

Centripetal force = ...75.... N **[3]**

5 Fig. 5 shows how the combined gravitational potential of the Earth and Moon varies with distance. A point X is marked on the graph.

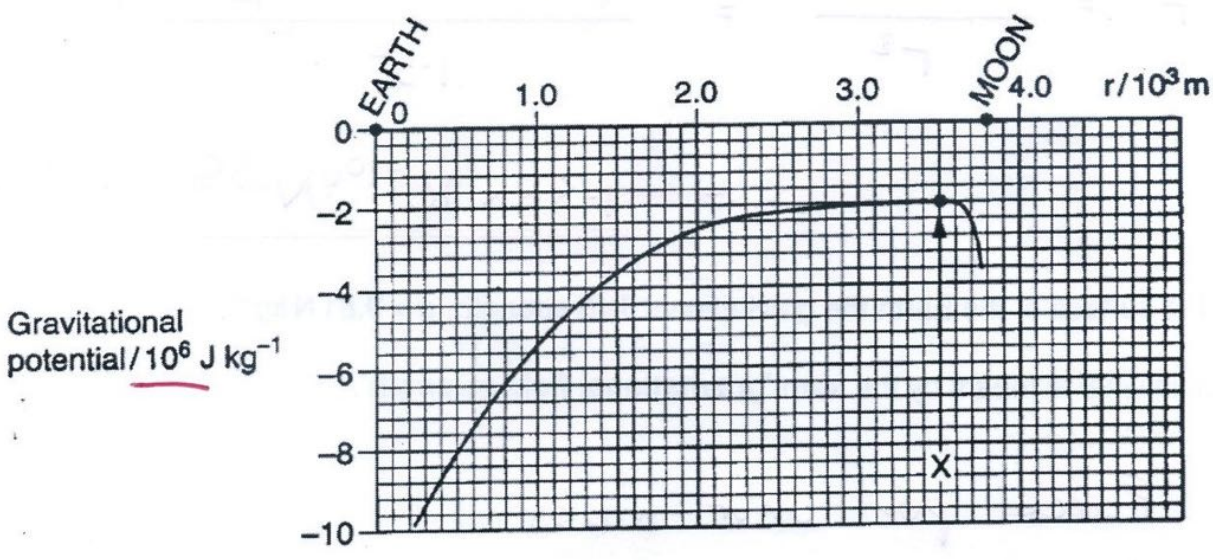


Fig. 5

(a) Use the graph to find the gravitational potential at point X.

Potential at X = ...-2×10^6.... J kg^{-1} **[1]**

(b) The gradient of the graph is zero at point X. This shows that the combined gravitational field strength of the Earth and Moon at that point is zero. Explain why point X is nearer the Moon than the Earth.

Mass of Moon < Mass of Earth so for the two* $\frac{GM}{r^2}$s to be equal and opposite r must be smaller for the Moon.

[2]

* one for Earth & one for Moon.

3 Two identical spheres are placed with their centres 1.5 m apart as shown. The mass of each sphere is 2.5 kg.

Show that the force of gravitational attraction between the spheres is 1.9×10^{-10} N. ($G = 6.67 \times 10^{-11}$ N m^2 kg^{-2})

$$F = \frac{(-)GMm}{r^2} = \frac{6.67 \times 10^{-11} \times 2.5^2}{1.5^2}$$

$$= \underline{\underline{1.85 \times 10^{-10}\ N}}$$

[2]

7 On the surface of the Earth the gravitational field strength, $g = 9.81$ N kg^{-1}.

Explain what is meant by the term **'gravitational field strength'**.

Force per unit mass

or

Gradient of V_{grav} at that point.

[2]

12 This question is about using airbags and seat belts to improve driver safety.

In a test laboratory, a car travelling at $11.0\,\mathrm{m\,s^{-1}}$ strikes a wall head-on and comes to rest in 0.1 s.

A crash test dummy of mass 75 kg is belted into the driver's seat of the car.

(a) Calculate the change of momentum of the dummy in the crash.

$\Delta p = m \Delta v = 75\,\mathrm{kg} \times 11.0\,\mathrm{ms^{-1}}$

change of momentum = ...825.... $\mathrm{kg\,m\,s^{-1}}$ [2]

(b) In the crash, the dummy is brought to rest by the seat belt from a speed of $11\,\mathrm{m\,s^{-1}}$, in a time of 0.14 s.

Show that the average force on the dummy is about eight times its weight.

$F = \dfrac{\Delta p}{\Delta t} = \dfrac{825\,\mathrm{kgms^{-1}}}{0.14\,\mathrm{s}} = 5893\,\mathrm{N}$

$W = mg = 75 \times 9.8 = 735\,\mathrm{N}$

$\dfrac{F}{W} = 5893/735 = \underline{\underline{8}}$

[2]

The seat belt does not stop the head of the dummy moving forward. With no airbag the head could strike the steering wheel. Fig. 12.1 shows how the force on the head of the dummy changes over time if the head strikes a steering wheel.

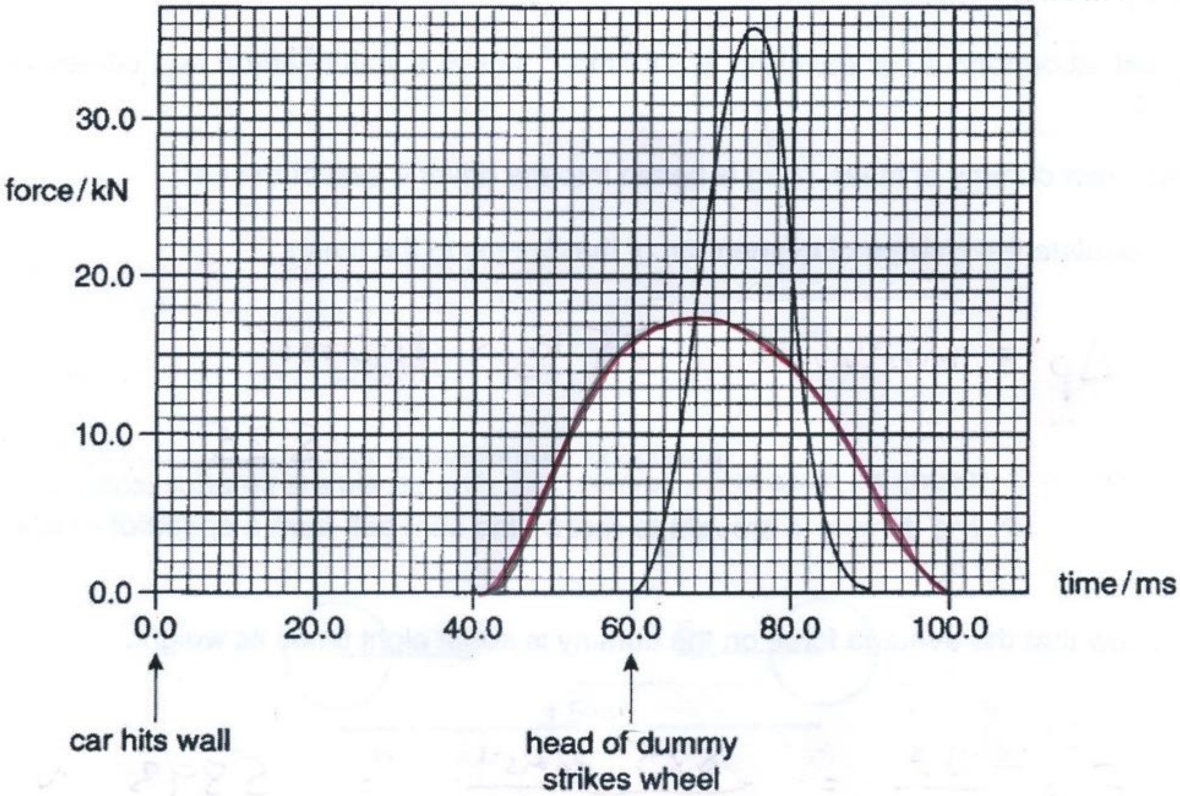


Fig. 12.1

If an airbag is installed it will begin to inflate from the steering wheel about 20 ms after the collision and takes a further 20 ms to fully inflate. 40 ms after the collision the bag begins to deflate.

(c) Suggest why airbags are most helpful if they are already deflating when the head strikes them.

If it is still inflating the impact velocity will be greater. It could push head back resulting in higher Δv and hence acc$^{\underline{n}}$ & hence force on head.

[2]

(d) Draw a second graph on Fig. 12.1 to show how the force on the head changes if an airbag is present.

Explain how your graph shows,

(i) that the average force on the head is lower with an airbag

peak is lower

(ii) that the change of momentum of the head is the same in both cases.

area is same

[4]

10 This question is about a new form of rocket engine called an Ion Drive that is used in the spacecraft Deep Space 1.
A stream of singly charged xenon ions enters the Ion Drive close to the anode. The ions are accelerated through a p.d. of 250 V and leave through a hole in the cathode as shown in Fig. 10.1.

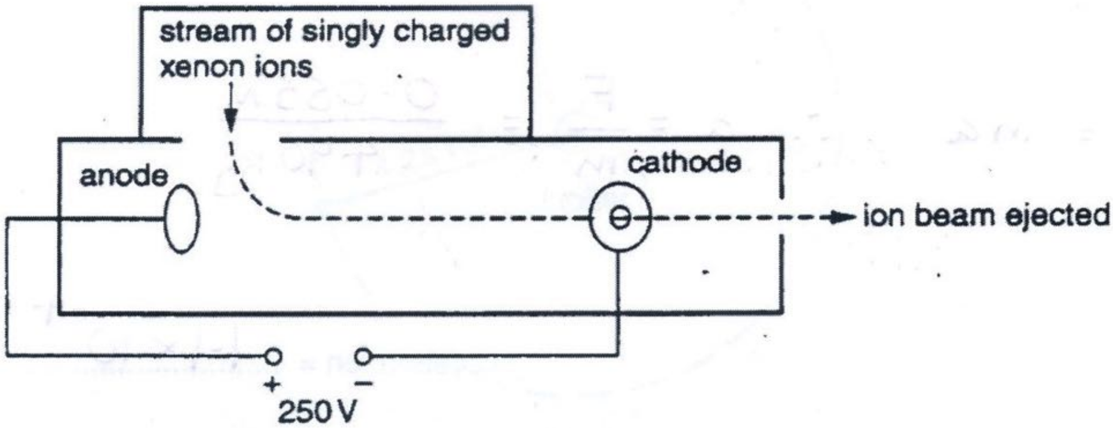


Fig. 10.1

(a) (i) Show that the kinetic energy gained by a single xenon ion is about 4×10^{-17} J when accelerated through a potential difference of 250 V.

$E = VQ = 250V \times 1.6 \times 10^{-19}$ J/V

$= \underline{4 \times 10^{-17} \text{ J}}$

(ii) The mass of a xenon ion is 2.2×10^{-25} kg. Show that the xenon ions leave the ion drive with a velocity of about 2×10^4 m s^{-1}.

$E_K = \frac{mv^2}{2} \therefore v = \sqrt{\frac{2E_K}{m}}$

$= \sqrt{\frac{2 \times 4 \times 10^{-17}}{2.2 \times 10^{-25}}} = \underline{1.91 \times 10^4 \text{ ms}^{-1}}$

[3]

(b) (i) The drive ejects 2.9×10^{-6} kg of xenon each second. Show that the momentum gained by this amount of gas passing through the ion drive is about 0.06 kg m s^{-1}.

$\Delta p = \Delta m v = 2.9 \times 10^{-6}$ kg $\times 1.9 \times 10^4$ ms^{-1}

$= \underline{0.055 \text{ N}}$

(ii) Explain why the force exerted on the spacecraft is about 0.06 N.

$F = \frac{\Delta p}{\Delta t} = \frac{0.055 \text{ kgms}^{-1}}{1\text{s}} = \underline{\underline{0.055 \text{ N}}}$

(iii) The spacecraft has a mass of 490 kg. Calculate the acceleration of the spacecraft.

$F = ma \quad \therefore \quad a = \frac{F}{m} = \frac{0.055\text{N}}{490 \text{ kg}}$

acceleration = 1.1×10^{-4} ms^{-2}

[5]

(c) Suggest and explain one possible effect of replacing the xenon ions with krypton ions, which have smaller mass but the same charge.

Since E_K will be the same & $E_K = \frac{mv^2}{2}$ then v will be greater. ($\Delta p/\Delta t$ & F will be lower as v increases by $\sqrt{\text{factor}}$ & m decrease by factor. where factor = mass Xe/mass Kr.)

[2]

4 A tennis ball of mass 0.11 kg travelling at 40 m s^{-1} hits a wall head on and bounces off, returning along the same path at 30 m s^{-1}. ie $-30 ms^{-1}$

(a) Calculate the change in velocity of the ball.

$-30 - 40 = -70 ms^{-1}$

prob not needed

change in velocity = -70 m s^{-1}

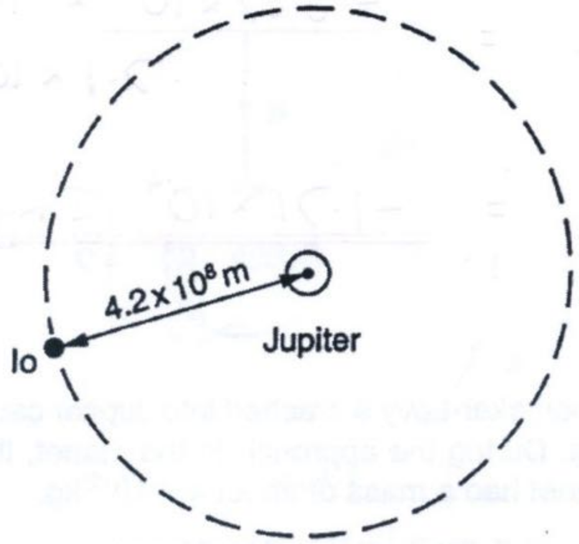
(b) Calculate the change in momentum of the ball. Include the unit in your answer.

$-70 ms^{-1} \times 0.11$ kg

prob not needed

change in momentum = -7.7 unit $kgms^{-1}$ [3]

9 This question is about the planet Jupiter and one of the moons that orbits it, called Io.



Io orbits Jupiter at a speed, v, of $1.7 \times 10^4\,m\,s^{-1}$ at an orbital radius, r, of $4.2 \times 10^8\,m$.

(a) Show that Io takes approximately 43 hours to orbit Jupiter once.

$$t = \frac{dist}{vel} = \frac{2\pi \times 4.2\times10^8}{1.7\times10^4} = 1.55\times10^5\,s$$

$$= \underline{\underline{43.1\text{ hrs}}}$$

[2]

(b) Io is held in its orbit by a centripetal force, $F = -\frac{mv^2}{r}$, where m is the mass of Io. This force is the gravitational attraction between Io and Jupiter.

(i) Show that $M = \frac{v^2 r}{G}$ where M is the mass of Jupiter.

$$\frac{-GMm}{r^2} = \frac{-mv^2}{r} \quad \therefore \quad \frac{GM}{r} = v^2$$

$$\therefore M = v^2 r / G$$

(ii) Show that the mass of Jupiter is about 2.0×10^{27} kg.
$G = 6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}$

$$M = \frac{v^2 r}{G} = \frac{(1.7\times10^4)^2 \times 4.2\times10^8}{6.67\times10^{-11}} = \underline{\underline{1.82\times10^{27}\text{ kg}}}$$

[4]

(c) Show that the gravitational potential at the top of Jupiter's atmosphere, 7.1×10^7 m from the centre of the planet, is about $-2 \times 10^9\,\mathrm{J\,kg^{-1}}$.

Assume that Jupiter is a sphere.

$$V_{grav} = \frac{-GM}{r} = \frac{-6.67\times10^{-11} \times 1.82\times10^{27}}{7.1\times10^{7}}$$

$$= \underline{\underline{-1.71\times10^{9}\ \mathrm{J\,kg^{-1}}}}$$

[2]

(d) In July 1994, comet Shoemaker-Levy 9 crashed into Jupiter causing dramatic heating of the planet's atmosphere. During the approach to the planet, the comet broke up. One piece that struck the planet had a mass of about 4×10^{12} kg.

This fragment crossed the orbit of Io heading directly towards Jupiter with a velocity of $10\,\mathrm{km\,s^{-1}}$.

(i) Show that the kinetic energy of the fragment at this moment is 2×10^{20} J.

$$E_K = \frac{mv^2}{2} = \frac{4\times10^{12} \times (10\times10^{3})^2}{2} = \underline{\underline{2\times10^{20}\ \mathrm{J}}}$$

[1]

(ii) Explain why the fragment will enter the atmosphere of Jupiter with a velocity greater than $10\,\mathrm{km\,s^{-1}}$.

E_{grav} lost = E_K gained and $E_K = mv^2/2$

so v must increase.

OR

Gravitational force accelerates comet fragment [2]

1 Fig. 1.1 shows three possible paths, **A, B** and **C**, of a spacecraft moving near the Earth but well above the atmosphere.

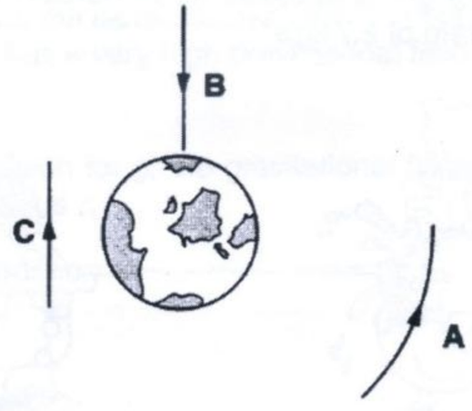


Fig. 1.1

(a) Which path follows a gravitational field line of the Earth ?

answer ....B........[1]

(b) Which path follows a gravitational equipotential line of the Earth?

answer ....A........[1]

6 A player serves a ball in a game of tennis with a racket of mass 0.35 kg. The racket moves with velocity $22\,m\,s^{-1}$ as it strikes the ball.

(a) Show that the momentum of the racket is about $8\,kg\,m\,s^{-1}$.

$$p = mv = 0.35\,kg \times 22\,ms^{-1} = \underline{\underline{7.7\,kgms^{-1}}}$$

[1]

(b) In a typical serve, about 25% of the momentum of the racket is transferred to the ball. The racket strikes a stationary tennis ball of mass 0.050 kg.

Estimate the velocity of the ball when it leaves the racket.

$$\Delta p = m\Delta v = 0.25 \times 7.7 = 1.93\,kgms^{-1}$$

$$\Delta v = \frac{\Delta p}{m} = \frac{1.93\,kgms^{-1}}{0.050\,kg} = \underline{\underline{39\,ms^{-1}}}$$

[2]

6 A circus clown fires a water gun that ejects water horizontally at a speed of $7.3\,m\,s^{-1}$. The water leaves the gun at a rate of $2.7\,kg\,s^{-1}$.

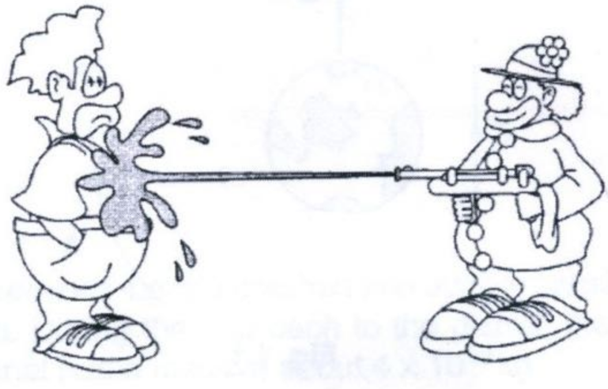


Fig. 6.1

(a) Show that the **rate of change of momentum** of the water on leaving the gun is about $20\,kg\,m\,s^{-2}$.

$$\frac{\Delta p}{\Delta t} = \frac{2.7 \times 7.3}{1} = 19.7\ kg\,m\,s^{-2}$$

[1]

(b) Explain why the clown holding the gun experiences a **backward force** of about 20 N.

Momentum is conserved so $\Delta p/\Delta t$ for water $= -\Delta p/\Delta t$ for clown & $\Delta p/\Delta t$ = Force

[2]

(c) The water strikes a second clown at the same velocity as it left the gun.

The water bounces off the clown.

Explain why this clown could experience a force **greater than** 20 N from the water jet.

Δv for water is greater as it bounces back

ends up with -ve velocity

and $F = \frac{m\,\Delta v}{\Delta t}$

[2]

9 **This question is about neutron stars.**

Neutron stars are the remnants of huge stars that have exploded as supernovae. One such neutron star has about the same mass as the Sun but its radius is only of the order of 10 km. Such a dense object has a very high gravitational field strength at its surface.

(a) (i) **Write down an expression for *g*, the gravitational field strength at the surface of a star of mass *M* and radius *r*.**

$$g = \frac{GM}{r^2}$$

[1]

(ii) **For a spherical star of average density ρ, the magnitude of *g* at its surface is given by**

$$g = \tfrac{4}{3}G\pi r\rho$$

where $G = 6.7 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.

Use this expression to show that the units of *g* are $\mathrm{N\,kg^{-1}}$.

$\frac{4}{3}$ G π r ρ

$Nm^2\,Kg^{-2} \cdot m \cdot Kgm^{-3}$

$= N \quad m^2.m.m^{-3} \quad Kg^{-2}.Kg = \underline{\underline{N\,Kg^{-1}}}$

[2]

(iii) **Show that the gravitational field strength at the surface of the star is about $1 \times 10^9\,\mathrm{N\,kg^{-1}}$.**

$\rho_{star} = 4.0 \times 10^{14}\,\mathrm{kg\,m^{-3}}$

$g = \frac{4}{3}G\pi r\rho = \frac{4}{3} \times 6.67 \times 10^{-11} \times \pi \times 10 \times 10^3 \times 4 \times 10^{14}$

$= \underline{\underline{1.1 \times 10^9\ N\,Kg^{-1}}}$

[2]

(b) A remarkable property of neutron stars is that they spin about their axes at a very great rate. The radiation from these stars is observed as regular pulses. This gives rise to the name 'pulsars'.

This particular neutron star of radius 10 km rotates 50 times every second.

(i) Show that the speed of a point on the equator of the star is about one percent of the speed of light.

$c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$

$$\text{speed} = \frac{\text{dist}}{\text{time}} = \frac{50 \times 2\pi r}{1} = 50 \times 2\pi \times 10 \times 10^3$$

$$= \underline{3.14 \times 10^6\,\mathrm{ms^{-1}}}$$

$$\frac{3.14 \times 10^6\,\mathrm{ms^{-1}} \times 100}{3 \times 10^8} = \underline{\underline{1.05\,\%}}$$

[2]

(ii) Calculate the magnitude of the centripetal acceleration at a point on the equator of the star. Include units in your answer.

$$a = \frac{v^2}{r} = (3.14 \times 10^6)^2 / 10 \times 10^3$$

acceleration 9.86×10^8 unit $\mathrm{ms^{-2}}$ [3]

(c) Neutron stars can spin at a great rate without flying apart because the gravitational field strength is high enough to keep material on the surface of the star.
Explain how this statement is supported by your answers to **(a)(iii)** and **(b)(ii)**.

$1.1 \times 10^9\,\mathrm{N\,kg^{-1}} > 9.86 \times 10^8\,\mathrm{ms^{-1}}$

$F/m = a > a$

gravitational acc$^{\underline{n}}$ > centripetal acc$^{\underline{n}}$

[3]

8 This question is about the physics of ejection seats.

Ejection seats are designed to fire an aircraft pilot out of the plane at high velocity.

One type of ejection system uses an explosion to accelerate the seat upwards.

The seat was tested in a plane standing on the runway (Fig. 8.1a).

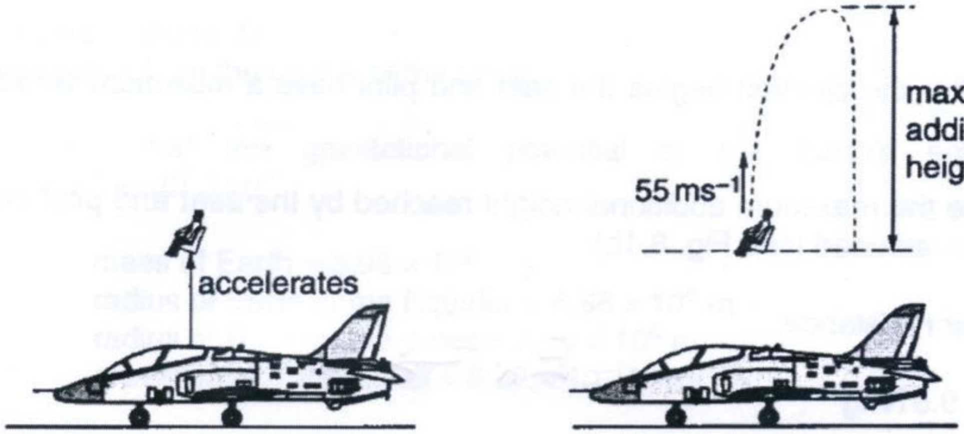


Fig. 8.1a **Fig. 8.1b**

The combined mass of the seat and pilot is 280 kg. When ejection takes place, the mass accelerates to a vertical velocity of 55 m s^{-1}.

(a) **(i)** Calculate the change of momentum of the seat and pilot.

$\Delta p = m\Delta v = 280\ kg \times 55\ ms^{-1}$

$= 1.54 \times 10^4\ kg\,ms^{-1}$

change of momentum = ... unit .... [3]

(ii) The change of momentum in (i) takes place in a time of 0.25 s.

Calculate the average force needed to give this change of momentum in 0.25 s.

$F = \frac{\Delta p}{\Delta t} = \frac{1.54 \times 10^4}{0.25 s} = 6.2 \times 10^4\ N$

force = ... N [2]

(iii) Suggest and explain how the body of the plane may move vertically during the ejection.

Δp for seat $= -\Delta p$ for plane so it will move down but not at such a high v as its mass is greater.

[2]

(b) 0.25 s after the ejection begins the seat and pilot have a maximum vertical velocity of $55\,\text{m s}^{-1}$.

Calculate the maximum additional height reached by the seat and pilot after maximum velocity is attained (see Fig. 8.1b).

Ignore air resistance.

$E_K \rightarrow E_{grav}$

$g = 9.8\,\text{N kg}^{-1}$

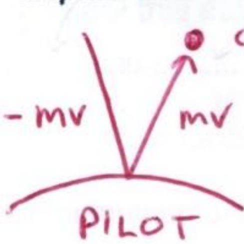
$$\frac{mv^2}{2} = mgh$$

$$h = \frac{v^2}{2g} = \frac{55^2}{2 \times 9.8} = \underline{\underline{154\,\text{m}}}$$

height = ... [3]

(c) The height reached after the ejection is less than this simple calculation suggests because of air resistance.

Explain how the particles of air exert a decelerating force on the pilot.



$\Delta p_{air} = 2mv$

so

$\Delta p_{pilot} = -2mv$

momentum transferred by collisions with air molecules will reduce total momentum of pilot & hence his/her velocity.

[3]

10 This question is about the gravitational field and potential near the Earth.

The gravitational potential V_{grav} due to the mass of an approximately spherical body is given by the expression

$$V_{grav} = \frac{-GM}{R}$$

where

M is the mass of the body
R is the distance from the centre of the body.

(a) (i) Show that the gravitational potential at the Earth's equator is about -6.25×10^7 J kg^{-1}.

mass of Earth = 5.98×10^{24} kg
radius of Earth at the Equator = 6.38×10^6 m
radius of Earth at the poles = 6.36×10^6 m
gravitational constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

$$V_{grav} = \frac{-GM}{R} = \frac{-6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.38 \times 10^6}$$

$$= -6.25 \times 10^7 \text{ J Kg}^{-1}$$

[2]

(ii) Show that the magnitude of the gravitational potential at one of the poles is about 1.003 times the magnitude at the equator.

$$V_{grav} \propto \frac{1}{R} \quad \therefore \text{ ratio} = \frac{6.38}{6.36} = 1.0031$$

[2]

(iii) Explain why gravitational potential is always negative.

Gravity is attractive so as radius falls from ∞ (which is defined as $V_{grav} = 0$) V_g falls as energy is lost from system.

[2]

(b) State an equation which shows how the gravitational field strength g outside a uniform sphere of mass M, varies with distance R from the centre of the sphere.

$$g = \frac{GM}{R^2}$$

[1]

(c) The strength of the gravitational field at the Earth's surface is not quite constant. There are local variations in field strength due to concentrations of dense material beneath the surface. The shape of the Earth also affects the field strength on the surface.

(i) Calculate the gravitational field strength at one of the Earth's poles.

$$g = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6.36 \times 10^{8})^2} =$$

gravitational field strength = 9.86 unit $N\,kg^{-1}$ [3]

(ii) The gravitational field strength at one of the poles is about 1.006 times the gravitational field strength at the equator.

The shape of the Earth changes the magnitude of the gravitational field strength by a factor of 1.006 but only changes the magnitude of the gravitational potential by a factor of 1.003.

Explain why these two factors are different.

$g \propto \frac{1}{r^2}$ and $V_{grav} \propto \frac{1}{r}$

& $1.003^2 = 1.006$

[2]

10 This question is about the time it takes a planet to orbit once around the Sun. This is called the **orbital period** of the planet.

In this question the following symbols will be used.

orbital period T
mean radius of orbit R
mass of sun M_s
mass of planet M_p

Fig.10.1

(a) In the seventeenth century Johannes Kepler (Fig. 10.1) suggested a relationship between the orbital period of a planet T and the radius of its orbit R. This relationship can be written as

$$T^2 \propto R^3.$$

Data for four of the planets are shown in Fig. 10.2.

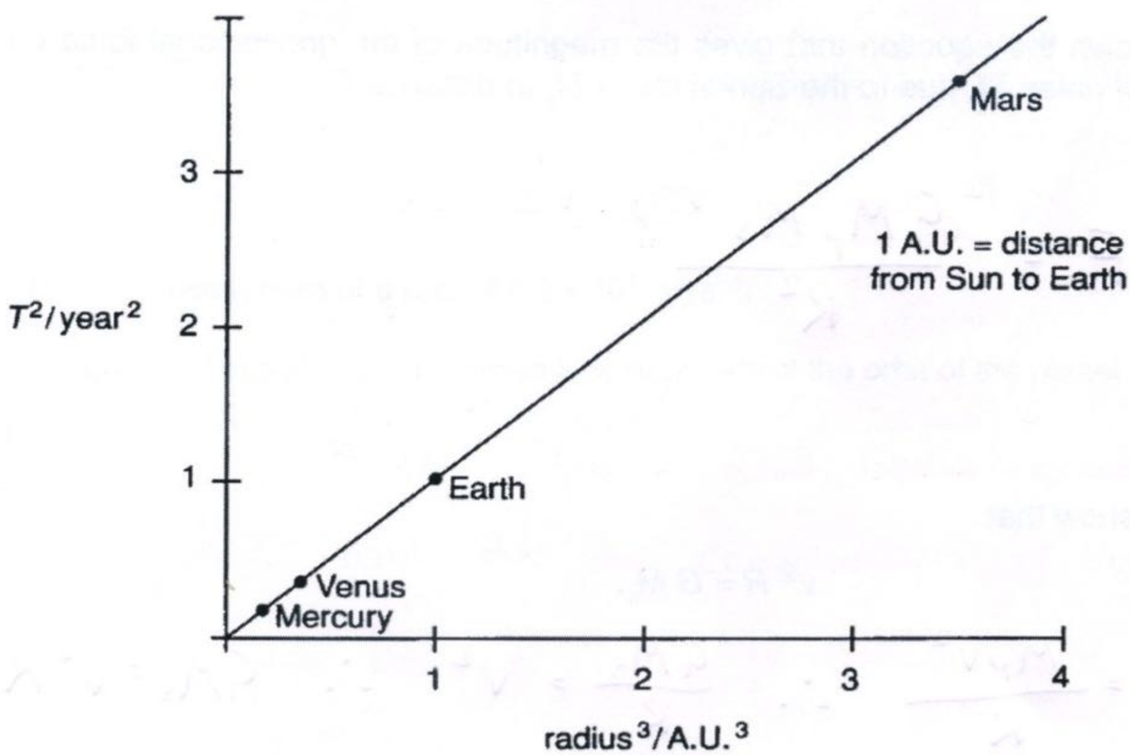


Fig. 10.2

State which features of the graph show that T^2 is proportional to R^3.

straight line through origin

[2]

(b) Isaac Newton (Fig. 10.3) developed a description of gravity that provided a mathematical backing for Kepler's work.

Fig. 10.3

(i) Suppose that a planet travels at speed v in a circular orbit of radius R. Using the equation for centripetal acceleration show that the centripetal force on the planet is given by the expression

$$F = \frac{M_p v^2}{R}.$$

[1]

(ii) Write down the equation that gives the magnitude of the gravitational force on a planet of mass M_p due to the Sun of mass M_s at distance R.

$$F = \frac{G M_p M_s}{R^2}$$

[1]

(iii) Hence show that

$$v^2 R = G M_s.$$

$$\frac{G M_p M_s}{R^2} = \frac{M_p v^2}{R} \quad \therefore \quad \frac{G M_s}{R} = v^2 \quad \therefore \quad G M_s = v^2 R$$

[1]

(iv) For a circular orbit, the speed v is given by

$$v = \frac{2\pi R}{T}.$$

Use this and the result of **(b)(iii)** to show that Newton could predict Kepler's law in the form

$$\frac{(2\pi)^2 R^3}{G M_s} = T^2.$$

$$GM_s = \left(\frac{2\pi R}{T}\right)^2 R \quad \therefore \quad GM_s = \frac{(2\pi)^2 R^2 R}{T^2}$$

$$\therefore \quad T^2 = \frac{(2\pi)^2 R^3}{G M_s}$$

[2]

(v) Calculate the mass of the Sun M_s using the following data for the orbit of the planet.

$G = 6.7 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$
$R = 1.5 \times 10^{11}\,\text{m}$
$T = 3.2 \times 10^{7}\,\text{s}$

$$M_s = 4\pi^2 R^3 / T^2 G$$

$$= \frac{4\pi^2 (1.5 \times 10^{11})^3}{(3.2 \times 10^7)^2 \times 6.67 \times 10^{-11}} =$$

mass of Sun = 1.95×10^{30} kg [2]

(c) The Sun loses mass at a rate of $6.2 \times 10^{11}\,\text{kg s}^{-1}$.

Suggest and explain how this mass loss might affect the orbit of the planet.

As M_s falls V_{grav} will become less negative and due to conservation of energy E_K must become less positive so v will fall. (Planet will move to a 'higher' orbit with larger R. in order to conserve angular momentum)

[2]