

Jan 2010

- 3 A small aircraft flies at a velocity of 200 km h^{-1} relative to the ground.

There is a wind blowing at 50 km h^{-1} from the west.

The pilot wishes to reach a destination due north of the starting point.

Find the resultant speed v of the aircraft, and the angle θ , west of north, which it must take.

Show your working clearly.

You may wish to draw a vector diagram.

$1 \text{ cm} = 20 \text{ km h}^{-1}$

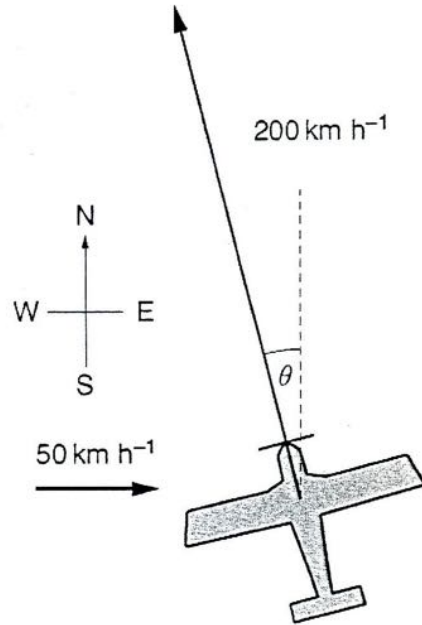


$V = \sqrt{200^2 - 50^2} = \underline{193.6 \text{ km h}^{-1}}$

OR $9.6 \text{ cm} \times 20 \text{ km h}^{-1} \text{ cm}^{-1} = \underline{192 \text{ km h}^{-1}}$

$\theta = \sin^{-1}\left(\frac{\text{opp}}{\text{hyp}}\right) = \sin^{-1}\left(\frac{50}{200}\right) = \underline{14.5^\circ}$

OR with protractor = 14.5°



$v = \underline{194} \dots \text{ km h}^{-1}$

$\theta = \underline{14.5} \dots^\circ \text{ W of N [3]}$

- 11 This question is about the vector nature of velocity and acceleration.
 At time $t = 0$, an object is moving in the x-direction at 5.0 ms^{-1} as shown in Fig. 11.1.
 Two seconds later, it is moving at 40° to that direction, but at the same speed.

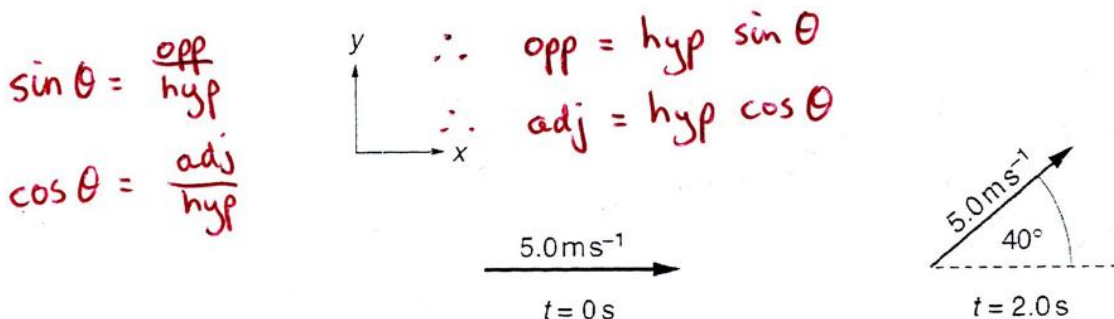
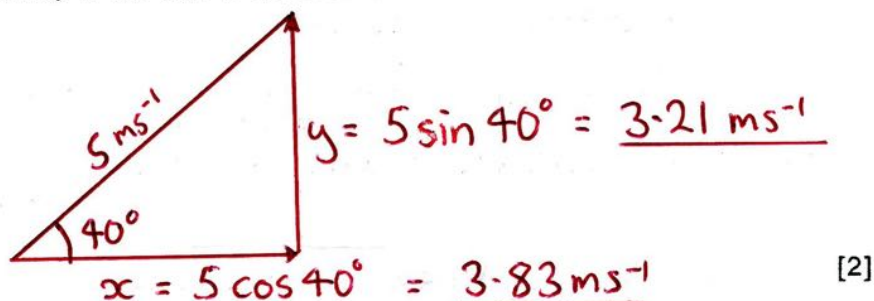


Fig. 11.1

- (a) (i) Show that the x-component of velocity at time $t = 2.0 \text{ s}$ is about 4 ms^{-1} and that the y-component of velocity at this time is about 3 ms^{-1} .



- (ii) Show that the mean x-component of acceleration during the 2.0 s is about -0.6 ms^{-2} .

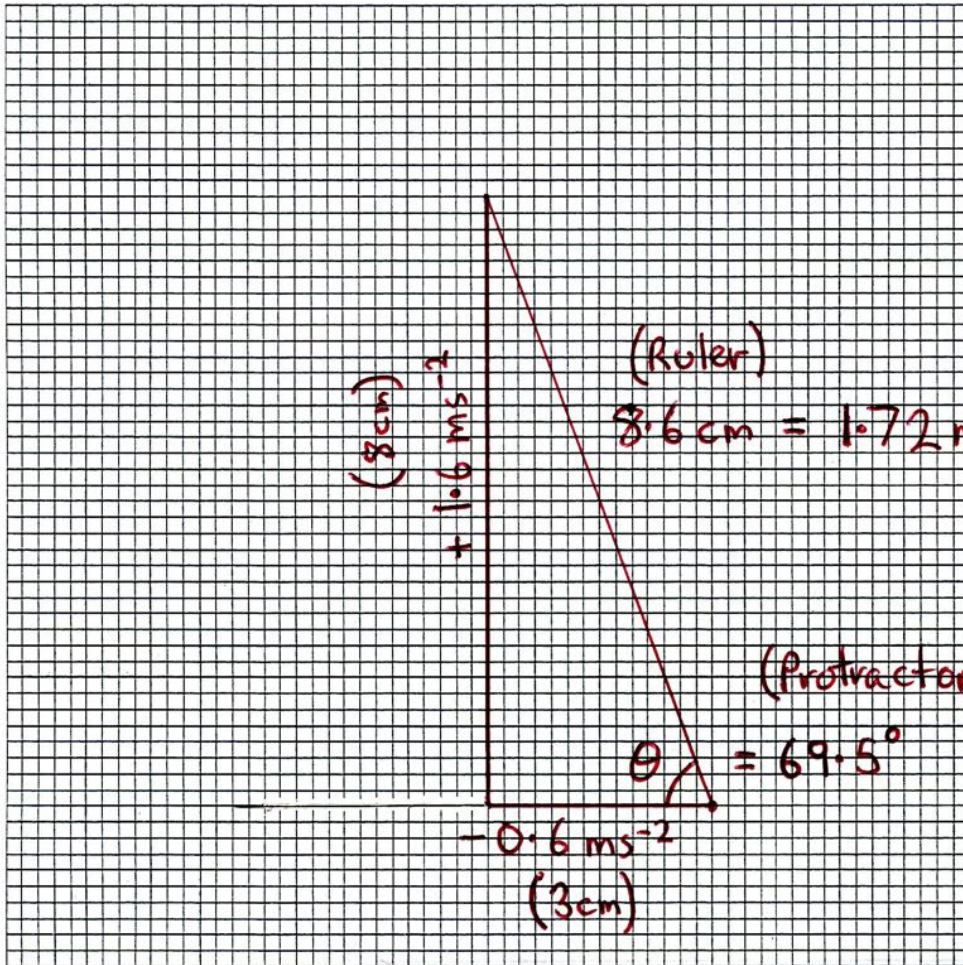
$$a = \frac{\Delta v}{\Delta t} = \frac{3.83 - 5.0}{2.0} = \frac{-1.17}{2.0} = \underline{\underline{-0.59 \text{ ms}^{-2}}}$$

[2]

- (b) The mean y-component of acceleration during the 2.0 s is $+1.6 \text{ ms}^{-2}$.

Choosing an appropriate scale, draw the two vector components of acceleration on the grid of Fig. 11.2 opposite and determine the magnitude and direction of the resultant acceleration.

magnitude of acceleration = 1.7 ms^{-2}
 direction of acceleration = 69° (from x-axis) $^\circ$



scale
 $\underline{1 \text{ cm} = 0.2 \text{ ms}^{-2}}$

→ +ve

↑ +ve

Fig. 11.2

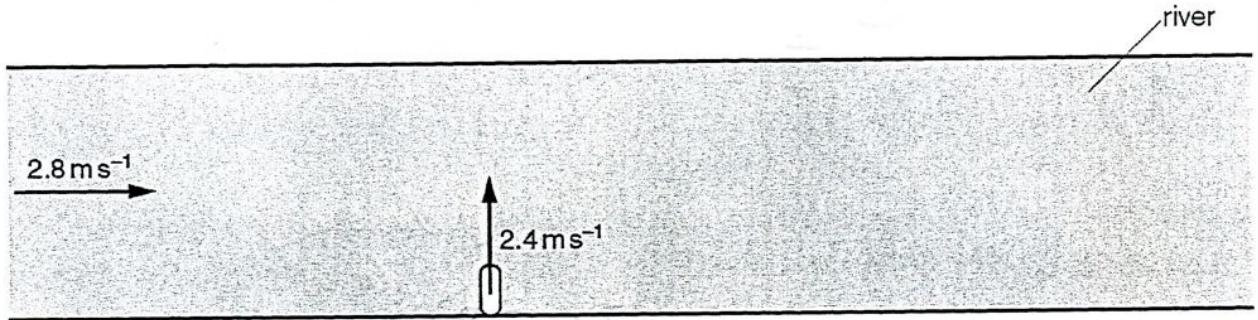
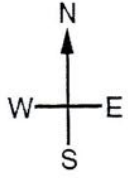
[4]

Using trig $a = \sqrt{1.6^2 + 0.6^2} = \underline{1.71 \text{ ms}^{-2}}$

$\theta = \tan^{-1} \frac{1.6}{0.6} = \underline{69.4^\circ}$

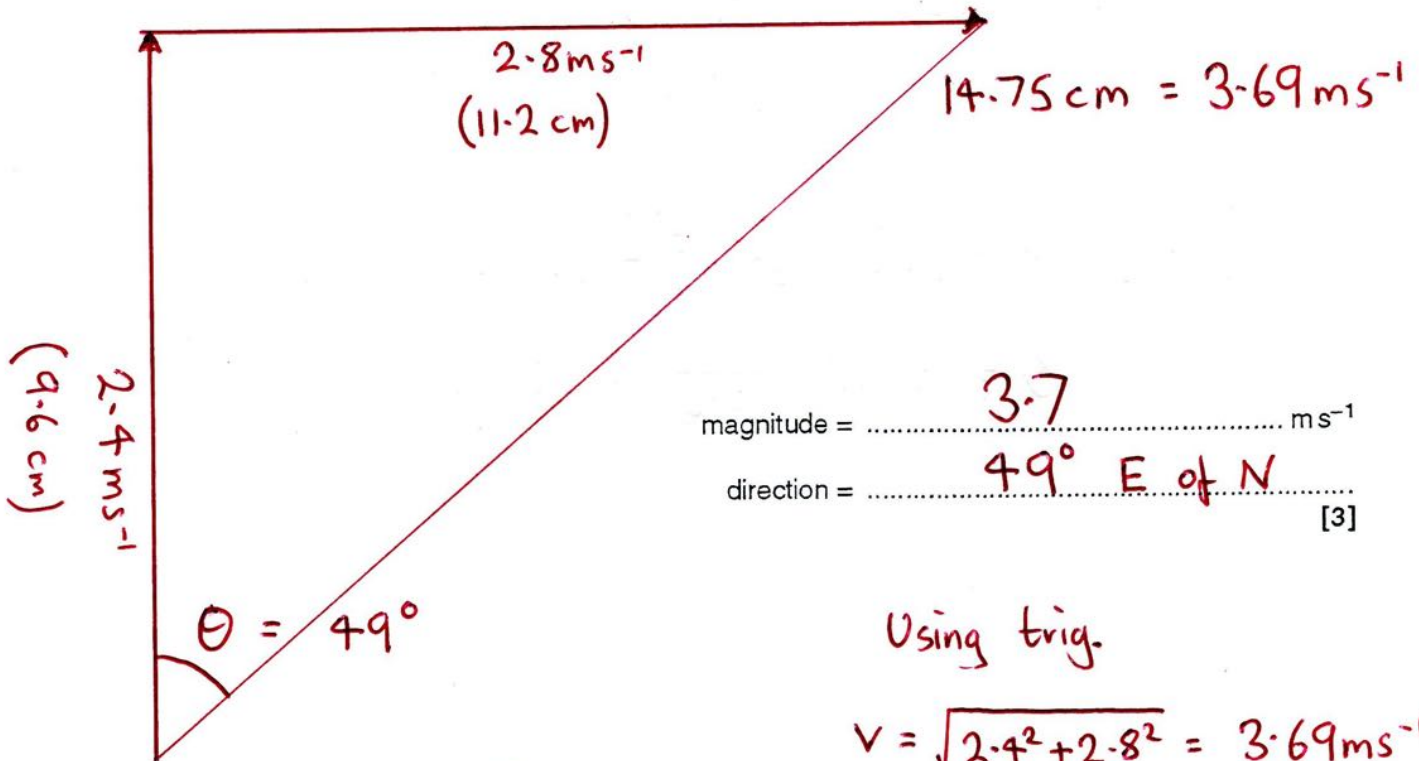
June 2015

5 A boat heads out north to cross a river as shown in the diagram.



The boat moves at 2.4 ms^{-1} in still water. The river is flowing due east at 2.8 ms^{-1} .

By scale drawing or by calculation, find the resultant velocity of the boat.



SCALE
1 cm = 0.25 ms^{-1}

Using trig.
 $v = \sqrt{2.4^2 + 2.8^2} = 3.69 \text{ ms}^{-1}$
 $\theta = \tan^{-1}\left(\frac{2.8}{2.4}\right) = 49.4^\circ$

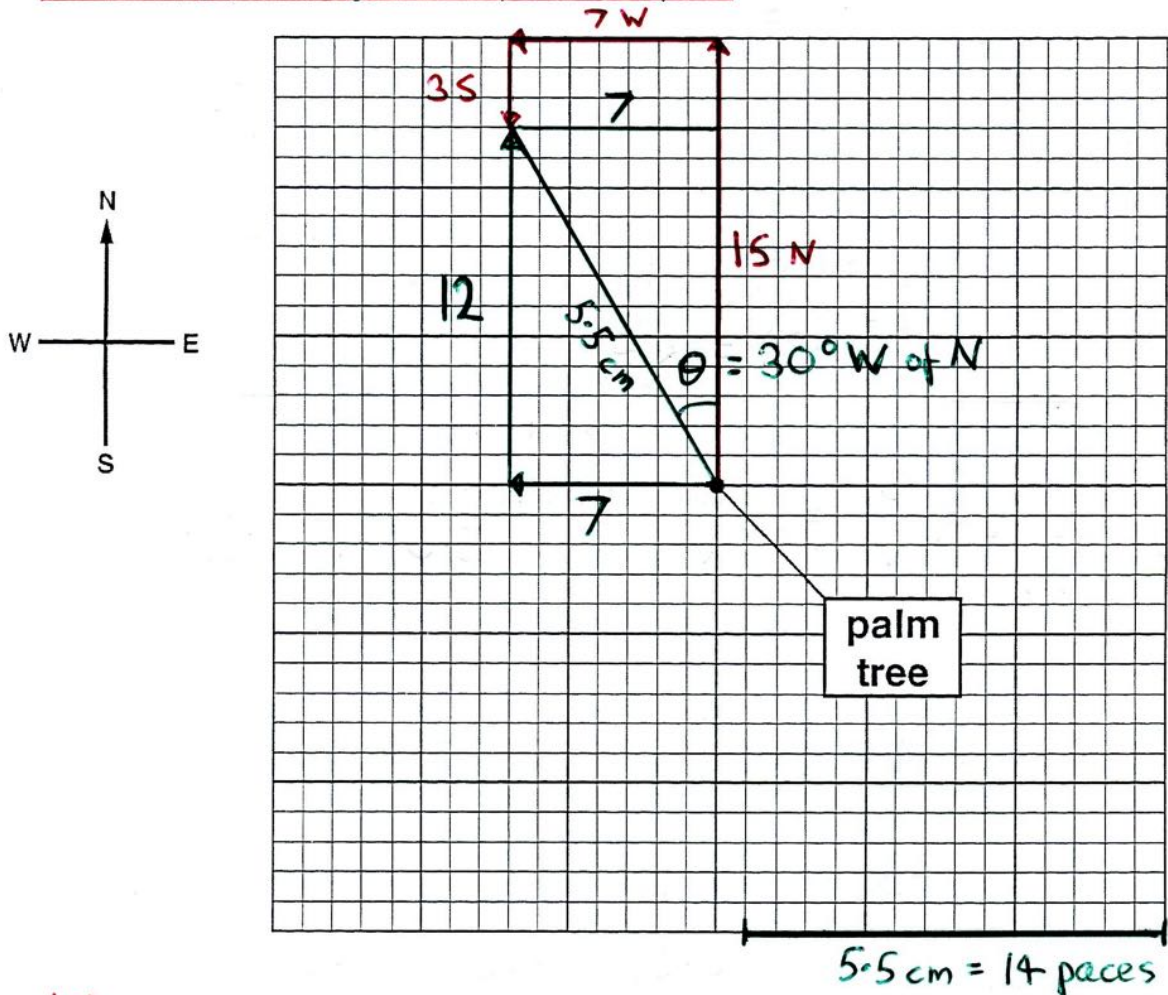
June 2011

- 7 A treasure map states:
- from the palm tree, go 15 paces north,
 - then go 7 paces west
 - the treasure is buried 3 paces south.

By calculation or drawing, find the magnitude and direction of the displacement of the treasure from the palm tree.

The central dot represents the palm tree.

Each small square on the grid below represents one pace.



Using trig.

$$S = \sqrt{12^2 + 7^2} = 13.9 \text{ paces}$$

$$\theta = \tan^{-1} \left(\frac{7}{12} \right) = 30.3^\circ$$

displacement = 14 paces
in a direction 30° W of N [3]

10 This question is about the vector nature of displacement, velocity and acceleration.

- (a) An object moves in the x - y plane along a semi-circular path from **A** to **C** as shown in Fig. 10.1. **B** is mid-way between **A** and **C**. The radius of the path is 3.0m and the object moves at a constant speed of 5.0ms⁻¹.

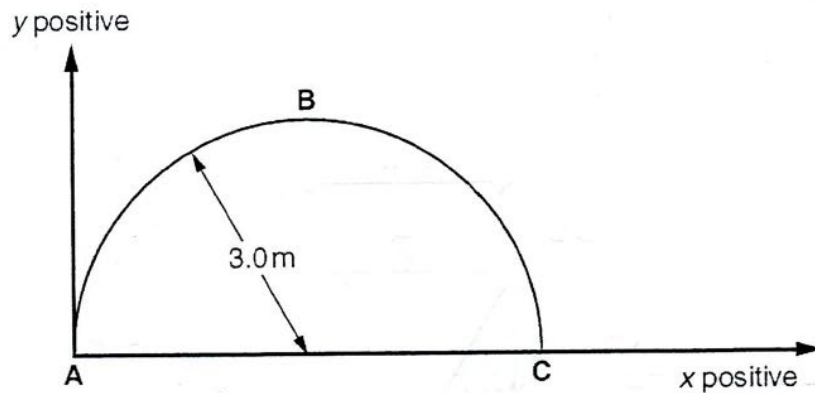


Fig. 10.1

- (i) Show that it takes about 2 seconds for the object to travel from **A** to **C**.

$$v = \frac{s}{t} \therefore t = \frac{s}{v} = \frac{\pi r}{v} = \frac{\pi \times 3.0}{5.0} = \underline{\underline{1.9 \text{ s}}}$$

[1]

- (ii) Write down the values of the x - and y -components of the **velocity** of the object when at **A**, **B** and **C** in the table below.

	velocity at A /ms ⁻¹	velocity at B /ms ⁻¹	velocity at C /ms ⁻¹
x -component	0.0	5.0	0.0
y -component	5.0	0.0	-5.0

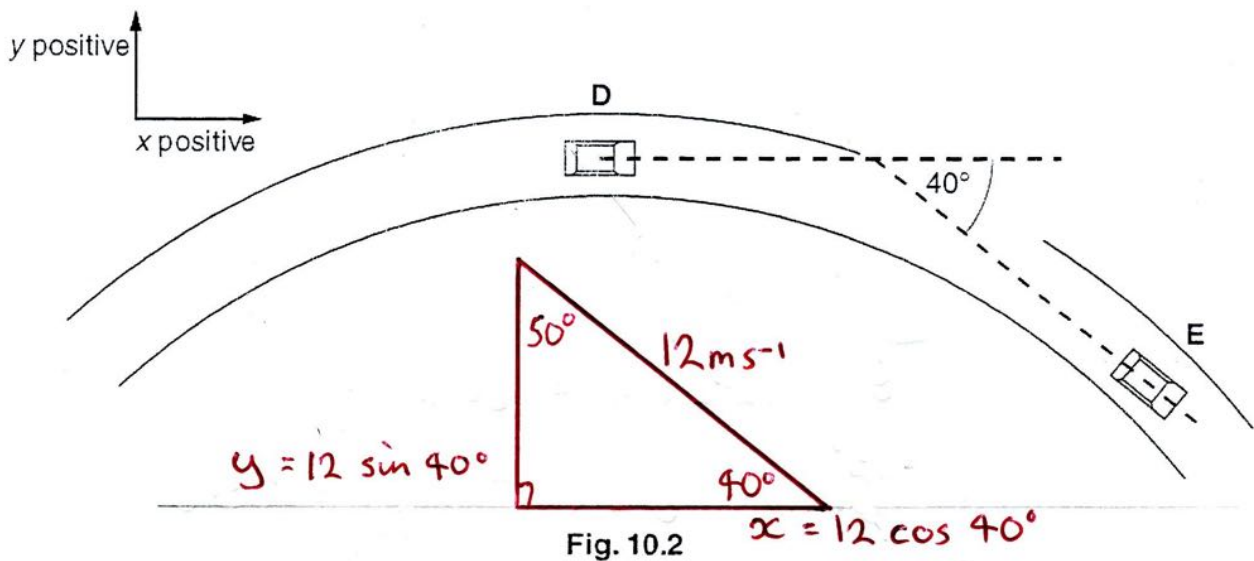
[2]

- (iii) Write down the values of the x - and y -components of the **displacement** of the object from **A** when at **B** and **C** in the table below.

	displacement from A to B /m	displacement from A to C /m
x -component	3.0	6.0
y -component	3.0	0.0

[2]

- (b) A car travels around a roundabout at a constant speed of 12 ms^{-1} . Its direction changes by 40° when moving from **D** to **E**, as shown in Fig. 10.2.



Because the velocity vector changes, the car has an acceleration. The car takes 1.6 s to travel from **D** to **E**. Calculate the mean values of the x- and y-components of acceleration between **D** and **E**. Show your working clearly.

x-component

$$\text{start } v = 12 \text{ ms}^{-1}$$

$$\text{end } v = 12 \cos 40^\circ = 9.19 \text{ ms}^{-1}$$

$$\Delta v = 9.19 - 12 = -2.81 \text{ ms}^{-1}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{-2.81}{1.6} =$$

$$\text{mean x-acceleration} = \dots -1.75 \dots \text{ms}^{-2}$$

y-component

$$\text{start } v = 0.0 \text{ ms}^{-1}$$

$$\text{end } v = -12 \sin 40^\circ = -7.71 \text{ ms}^{-1}$$

$$\Delta v = -7.71 - 0 = -7.71 \text{ ms}^{-1}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{-7.71}{1.6} =$$

$$\text{mean y-acceleration} = \dots -4.82 \dots \text{ms}^{-2}$$

[4]

6 An aircraft is in level flight through still air.

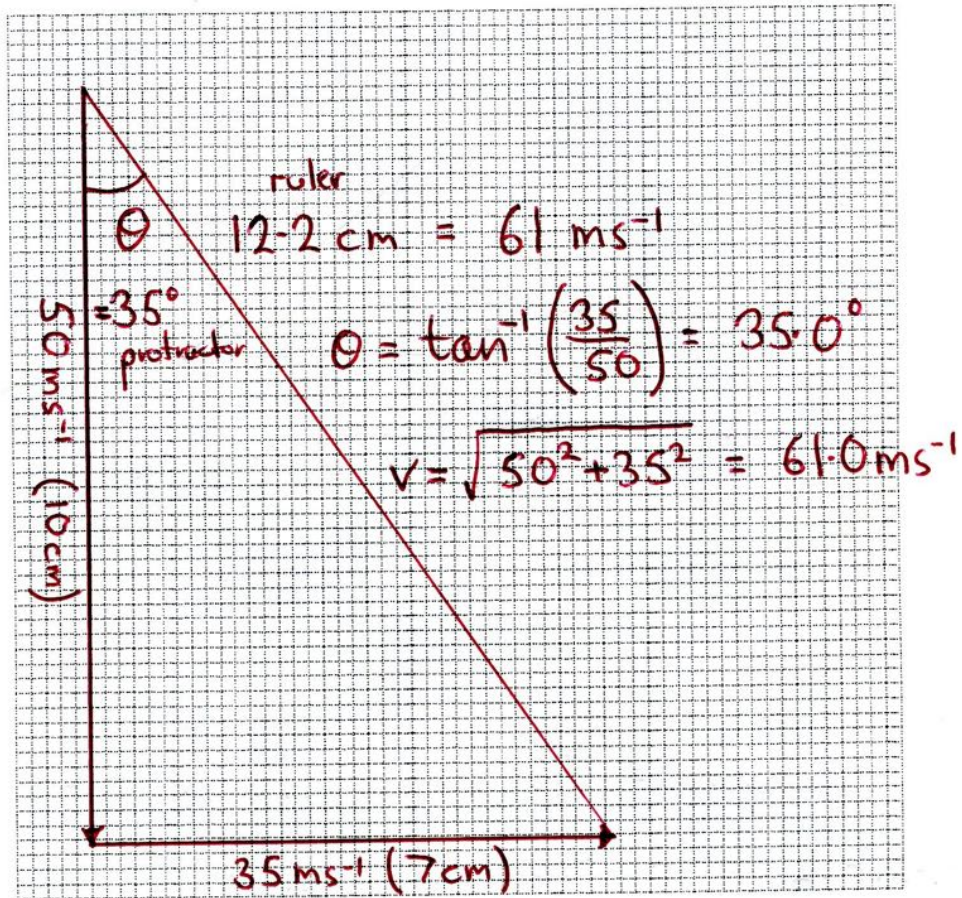
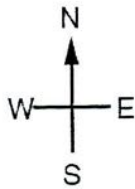
SCALE

It has the following components of velocity:

north-south: 50 ms^{-1} towards the south
 east-west: 35 ms^{-1} towards the east

$1 \text{ cm} = 5 \text{ ms}^{-1}$

- (a) Using a suitable scale, draw these components on the grid below. Hence or otherwise find the speed and direction of flight of the aircraft.



speed = 61 ms^{-1}

direction = 35° E of S [4]

OR bearing of 145°

- (b) A wind later acts upon the aircraft, changing the aircraft's direction so that it travels due south. Deduce the speed and direction of this wind.

Wind must cancel out the E-W component of the aircraft's velocity.

speed = 35 ms^{-1}

direction = towards the west [2]

OR bearing of 270°