13.3 **Special Relativity**
Name SA

Past Paper Questions

- Describe and explain the use of radar-type measurements to determine distances within the solar system; how distance is measured and defined in units of time, assuming the relativistic principle of the invariance of the speed of light.
- Describe and explain effect of relativistic time dilation using the relativistic factor $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

G494 Jan 2010

4 The relativistic time dilation factor γ is given by $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$.

(a) Show that the value of γ for a particle moving in a beam at a relative speed of $2.0 \times 10^8\,\mathrm{m\,s^{-1}}$ is about 1.3.

$c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$

$$\gamma = 1 \Big/ \sqrt{1 - \frac{(2\times10^8)^2}{(3\times10^8)^2}} = 1 \Big/ \sqrt{1 - \left(\frac{2}{3}\right)^2} = 1.342$$

[1]

(b) The particle is unstable and decays with a half-life $T_{½}$ of 8.2×10^{-7} s when it is at rest. Calculate the observed half-life of the particles moving in the beam.

$$= 8.2\times10^{-7}\,\mathrm{s} \times 1.342 =$$

$T_{½}$ = 1.10×10^{-6} s **[1]**

G494 June 2015

3 The half-life of π^+ mesons at rest in a laboratory is 18 ns. When a beam of fast-moving π^+ mesons move through the laboratory their measured half-life becomes 42 ns.

By calculating the relativistic factor γ for the π^+ mesons in the beam, determine their speed v through the laboratory.

$c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$

$$\gamma = \frac{42}{18} = 2.333$$

$$2.333 = 1 \Big/ \sqrt{1 - v^2/c^2} \quad \therefore \left(1/2.333\right)^2 = 1 - v^2/c^2 = 0.184$$

v = 2.7×10^8 m s^{-1} **[3]**

$$\therefore \frac{v}{c} = \sqrt{1 - 0.184} = 0.904$$

$$\therefore v = 0.904c = \underline{\underline{2.71\times10^8\,\mathrm{ms^{-1}}}}$$

G494 Jan 2012

12 Fig. 12.1 shows the worldline of a spacecraft which passes the Earth and then returns.

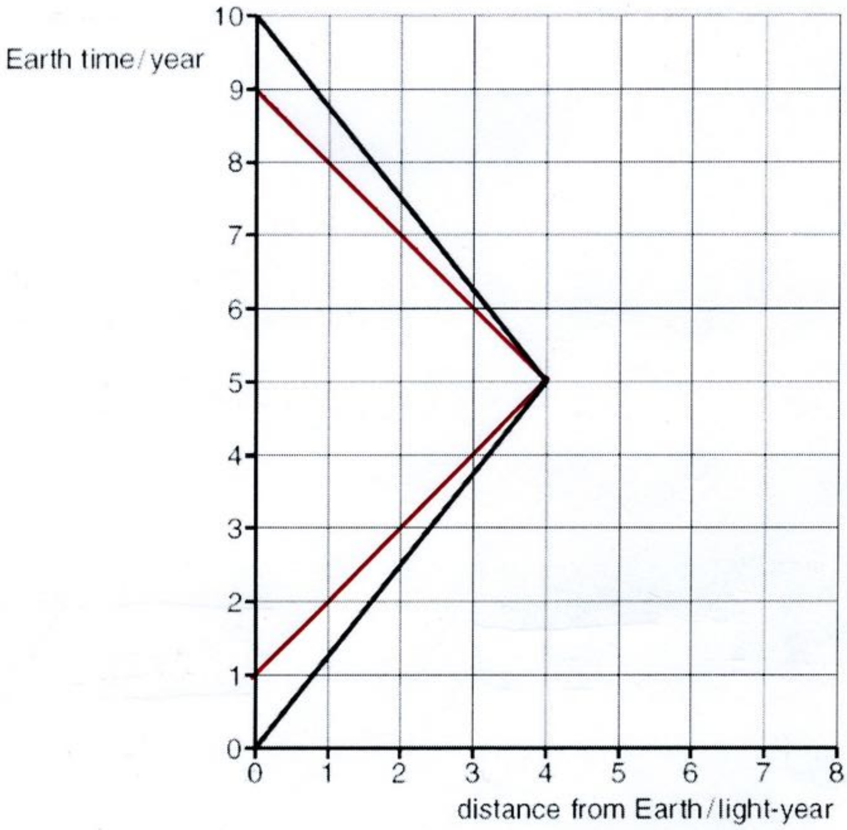

Fig. 12.1

Clocks on the Earth and spacecraft are zeroed at the instant that the spacecraft passes the Earth.

(a) The worldline for the spacecraft is a straight line until $t = 5$ year.
What does this tell you about the motion of the spacecraft?

It is constant

[1]

(b) A single pulse of light is sent towards the spacecraft from the Earth when the Earth clock reads $t = 1.0$ year. It reflects off the spacecraft and returns to Earth.

(i) Why is the worldline for light always at 45° on Fig. 12.1?

Light travels 1 light-year each year and the graph as the same size division for light-years and years.

[1]

(ii) Draw the complete worldline of the pulse of light on Fig. 12.1. **[2]**

(c) The arrival of the pulse of light at the spacecraft is the signal for it to turn around and return to the Earth.

(i) Explain how an observer on Earth can use the times of emission and reception of the pulse to calculate that the spacecraft was 4.0 light-year from the Earth when the pulse reached it.

The pulse took $9y - 1y = 8$ years to return so distance must be half that. $8/2 = 4$ light-years

[2]

(ii) Explain how an observer on the Earth can use the time of emission and return of the pulse to deduce that the spacecraft turned round when the Earth clock reads $t = 5.0$ year.

The pulse must have taken 4 years to reach the craft (out and return trips are of equal length) and it left 1 year before. $1 + 4 = 5$ years

[2]

(iii) Show that the outward speed of the spacecraft relative to the Earth is $2.4 \times 10^8 \, m\,s^{-1}$.

$c = 3.0 \times 10^8 \, m\,s^{-1}$

$$v = s/t = 4\,ly/5y = 0.8\,c$$

$$0.8 \times 3 \times 10^8 = 2.4 \times 10^8 \, ms^{-1}$$

[1]

(d) **(i)** Show that the time dilation factor γ for a spacecraft travelling relative to the Earth at velocity $v = 2.4 \times 10^8 \, m\,s^{-1}$ is about 1.7.

$$\gamma = 1/\sqrt{1 - v^2/c^2} = 1/\sqrt{1 - 0.8^2} = 1.67$$

[1]

(ii) Here are some possible times in year for the round trip according to observers on the spacecraft. Put a ring around the correct value.

6.0 **8.0** **10** **17**

[1]

$$10 \text{ years}/1.67 = 6 \text{ yrs}$$

(Faster = slower clocks $\therefore$ less time)

G494 June 2016

8 The graph of Fig. 8.1 shows a time-distance graph of an unstable particle as it passes through a laboratory.

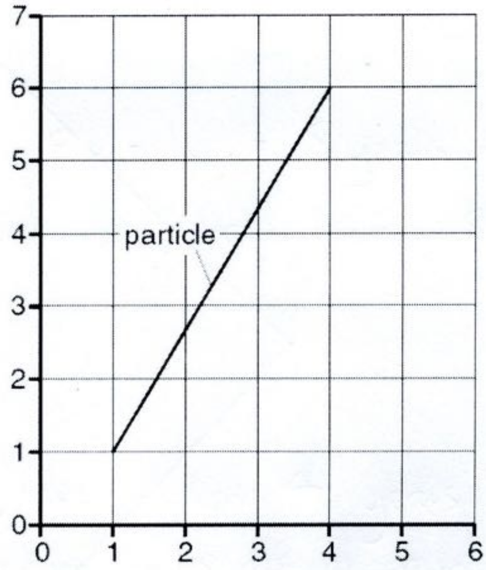

Fig. 8.1

The particle is produced at time 1.00 ns, measured by a clock in the laboratory.
The particle decays at time 6.0 ns, giving it a life time of 5.0 ns.

(a) Show that the speed of the particle through the laboratory is about $2 \times 10^8\,\text{m s}^{-1}$.
$c = 3.0 \times 10^8\,\text{m s}^{-1}$

$$v = s/t = \frac{3 \text{ light-ns}}{5 \text{ ns}} = \frac{3 \times 10^8 \times 3 \times 10^{-9}}{5 \times 10^{-9}} = 1.8 \times 10^8\,\text{ms}^{-1}$$

[1]

(b) Calculate the relativistic factor γ, and determine the life time of this particle if it had been at rest in the laboratory.

$$\gamma = 1 \Big/ \sqrt{1 - v^2/c^2} = 1 \Big/ \sqrt{1 - 0.6^2} = 1.25$$

At rest $t_{\frac{1}{2}}$ must be shorter $\therefore$ $\frac{5 \text{ ns}}{1.25} = t_{\frac{1}{2}}$

$$\frac{1.8 \times 10^8}{3 \times 10^8} = 0.6$$

life time = ...4.0...... ns **[2]**

G494 June 2017

6 Muons have a half-life of about 1.5 μs at rest. The observed half-life of muons produced from cosmic rays is about 7.5 μs.

(a) Calculate the relativistic factor γ of the muons produced from cosmic rays.

$$\gamma = 7.5/1.5 = 5$$

relativistic factor = 5 [1]

(b) Calculate the speed of the muons produced from cosmic rays.

$c = 3.0 \times 10^8 \mathrm{m\,s^{-1}}$

$$5 = 1/\sqrt{1 - v^2/c^2} \quad \therefore \left(1/5\right)^2 = 1 - v^2/c^2$$

$$v^2/c^2 = 1 - \left(\frac{1}{5}\right)^2 \quad \therefore \quad v/c = \sqrt{1 - \left(1/5\right)^2} = 0.980$$

$$\therefore v = 3 \times 10^8 \times 0.980$$

speed = 2.94×10^8 m s⁻¹ [2]

OR use

$$\frac{v}{c} = \sqrt{1 - 1/\gamma^2} = \sqrt{1 - 1/5^2} = 0.980$$

Specimen 557/01

34 The half-life of a muon at rest is 1.52 μs. Muons in cosmic rays are observed to have half-lives of 10.4 μs.

Calculate the velocity of the muons in cosmic rays.

$\gamma = 10.4/1.52 = 6.842$

$\therefore 6.842 = 1/\sqrt{1 - v^2/c^2} \quad \therefore \left(1/6.842\right)^2 = 1 - v^2/c^2 =$

$= 0.0214$

$\therefore v/c = \sqrt{1 - 0.0214} = 0.989c$

$\therefore v = 3\times10^8 \times 0.989 =$ velocity = 2.97×10^8 m s^{-1} **[3]**

30 The relativistic factor $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$

Which statement about this factor is correct?

A At the speed of sound γ is close to ~~zero~~ one. ✗

B $\gamma \to$ ~~1~~ ∞ as $v \to c$. ✗

C γ predicts the time dilation factor so that moving clocks run slower as $v \to c$. ✓

D γ^2 is the factor by which the total energy of a moving particle is greater than its rest energy. ✗

$E_t = \gamma E_{rest}$

Your answer C **[1]**

H557/02 2017

6 This question is about muon decay. Muons are charged leptons. They are formed by cosmic rays interacting with the upper atmosphere.

The decay equation of a negative muon, μ^- is:

$$\mu^- \rightarrow e^- + \overline{\nu} + \nu$$

where $\overline{\nu} + \nu$ represent an antineutrino and a neutrino respectively.

Covered in other chapter.

(c) Muons travel through the atmosphere at 98% of the speed of light.
The half-life of a muon at rest is about 1.5×10^{-6} s. Show that about 0.0005% of the original muons will remain after travelling 8 km through the atmosphere, ignoring relativistic effects.

$V = 0.98c = 0.98 \times 3 \times 10^8 = 2.94 \times 10^8 \text{ ms}^{-1}$

$t = s/v = 8 \times 10^3 / 2.94 \times 10^8 = 2.72 \times 10^{-5} \text{ s}$

No of half-lives $= 2.72 \times 10^{-5} / 1.5 \times 10^{-6} = 18.1$

Fraction remaining $= 1/2^{18.1} = 3.46 \times 10^{-6}$

$= 0.00035\ \%$

[3]

(d) (i) In a measurement it is found that about 9% of the muons remain after travelling through 8 km of atmosphere.
Explain why a greater number of muons remain than suggested by the non-relativistic calculation in **(b)**.

The muons are moving at close to the speed of light so the relativistic factor γ will be larger than 1 so the half-life will be time dialated as $t = \gamma \tau$. [3]

(ii) Use your answer to **(c)** and the measured value of 9% of muons remaining after passing through 8 km of atmosphere to calculate the relativistic factor γ for the muons.

$9\% = 0.09$

$\therefore \frac{1}{2^{HL}} = 0.09 \quad \therefore 2^{HL} = 1/0.09$

$\therefore HL = \log_2 (1/0.09) = 3.47$ half-lives

$\gamma = 18.1/3.47 =$

relativistic factor γ = 5.2 [3]