## Understanding Processes

Chapter 7: Quantum Physics
T test booklet

Name:

Target Grade :

| Date | T test | $\%$ | Corrections made |
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## Progress tracker

Shade in squares to indicate your level of performance in each test. Do this in the order they were taken.
(for the purposes of this grades are based on \% marks where A* 90, A 80 , B 70 etc..)

If you have to shade off the chart it is hopefully above rather than below

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| TG-2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## t test : Quantum Physics 1

$4 \quad$ Fig. 4.1 shows a portion of the electromagnetic spectrum on a logarithmic scale of photon energy.


Fig. 4.1
Using Fig. 4.1, write down
(a) the name of the region labelled P...............................................................[1]
(b) a photon energy in the visible region
$5 \quad$ Fig. 5.1 shows two rotating phasors at a time $\mathrm{t}=\mathrm{O}$. The phasors rotate in an anticlockwise direction.


Fig. 5.1
Draw on Fig. 5.1
(a) two phasors showing the positions at $t=1 / 4$ period,
(b) two waveforms, labelled A and B, for which the phasor diagrams are appropriate.

6 An LED emits red light of frequency $5.0 \times 10^{14} \mathrm{~Hz}$.
(a) Calculate the energy of a photon of red light.
(b) Calculate the number of photons of red light emitted per second when the radiated power of the LED is 2.0 mW .
number of photons per second $=$
3 Fig. 3.1 shows a graph of two waves $A$ and $B$.


Fig. 3.1
(a) State the phase difference between the two waves A and B .
$\qquad$
(b) Draw on Fig. 3.1 phasors to represent $A$ and $B$ at time $t$. Assume that the phasors rotate in an anticlockwise direction.

## t test: Quantum Physics 2

6 Photons travel from a source $S$ to detectors $D$ and $E$ through three equally spaced slits (Fig. 6.1).


Fig. 6.1
(a) Draw in the space below three phasors associated with the paths from the three slits for the case where $D$ is at a bright maximum of intensity.
(b) At detector $E$ the three phasors are $\uparrow \rightarrow \downarrow$.
(i) Draw the resultant of these three phasors.
(ii) State the ratio of the amplitudes at D and E .

4 Radiowaves of wavelength $\lambda=1500 \mathrm{~m}$ are all around us. They carry a radio programme into our homes.
(a) Calculate the frequency of a radiowave photon.

$$
c=3.0 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}
$$

(b) High frequency X-rays of frequency $f=2.4 \times 10^{17} \mathrm{~Hz}$ are used in hospitals to obtain images of the internal structure of the body.

Calculate the value of the ratio

$$
\frac{\text { energy of an X-ray photon }}{\text { energy of a radiowave photon }} \text {. }
$$

the Planck constant $h=6.6 \times 10^{-34} \mathrm{~J} \mathrm{~s}$
ratio =

3 (a) Calculate the energy of a photon of light of frequency $f=6.0 \times 10^{14} \mathrm{~Hz}$.
The Planck constant $h=6.6 \times 10^{-34} \mathrm{~J}$ s.
energy =
(b) A light-sensitive cell on the retina at the back of the eye will respond when $1.1 \times 10^{-18} \mathrm{~J}$ of light energy falls on it.

How many photons of light, frequency $f=6.0 \times 10^{14} \mathrm{~Hz}$, are required to trigger the response of a single cell?

## t test: Quantum Physics 3

10 This question is about a reflecting telescope.
When observing a distant galaxy, 180 photons of visible light are incident on one pixel of the telescope detector (Fig. 10.1).


Fig. 10.1
(a) (i) Visible light has an average wavelength of 550 nm . Calculate the average energy of each photon detected.
energy =
(ii) Calculate the total energy which fell on this pixel during the observation.
(iii) The observation lasted 1.0 hour. In each second during this observation, what is the probability of a photon arriving on this pixel?
probability $=$ $\mathrm{s}^{-1}$
(b) The telescope has a parabolic mirror to concentrate light on the detector placed at the focus (Fig. 10.2).


Fig. 10.2
(i) Using the wave point view, plane waves arrive at the mirror. Sketch on Fig. 10.2 wave fronts travelling back from the mirror.
(ii) From the point of view of quantum behaviour, a single photon arrives at the focus via the mirror, with a certain probability. Use ideas of quantum behaviour of a photon to explain how the shape of the mirror is designed to optimise the probability of arrival of a photon at the detector.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## t test : Quantum Physics 4

11 This question is about the emission of electrons from a metal surface.
A thin, square specimen of metal, dimensions $4.2 \times 10^{-2} \mathrm{~m} \times 4.2 \times 10^{-2} \mathrm{~m}$, is placed on the bench. The specimen is uniformly illuminated from above by a beam of electromagnetic radiation of frequency $6.0 \times 10^{15} \mathrm{~Hz}$, as shown in Fig.11.1.


Fig. 11.1
Fig. 11.2

Assume that the atoms in the metal surface are in a simple regular arrangement, as shown in Fig. 11.2.
(a) (i) Show that there are $1.5 \times 10^{8}$ atoms along the edge $X Y$ of the specimen in the surface layer.
diameter of an atom $=2.8 \times 10^{-10} \mathrm{~m}$.
(ii) Show that there are about $2.3 \times 10^{16}$ atoms in the whole surface.
(b) (i) Every second, $9.0 \times 10^{-7} \mathrm{~J}$ of energy is incident on the metal surface. Assume the energy arrives continuously and is completely absorbed by the atoms in the surface layer. Show that, on average, the amount of energy absorbed every second by each atom in the metal surface is $4.0 \times 10^{-23} \mathrm{~J}$.
(ii) The energy required to remove an electron from an atom of the metal is known to be $3.2 \times 10^{-18} \mathrm{~J}$.
Calculate the time taken for an atom in the metal surface to absorb this energy from the electromagnetic radiation.

$$
\text { time }=
$$

The experimental result is quite different. When electromagnetic radiation falls on the metal surface, some electrons are emitted immediately from the surface. This is one crucial result that indicates that photoelectric emission cannot be explained if the energy is assumed to arrive continuously.
(c) The quantum theory assumes that electromagnetic radiation of frequency $f$ is absorbed in discrete packets of energy (photons), each of energy $E=h f$.

Show that when a photon of electromagnetic radiation, frequency $f=6.0 \times 10^{15} \mathrm{~Hz}$, is absorbed by an atom of the metal, emission of an electron could occur.
t test : Quantum Physics 5
9 This question is about photon energies.
(a) A powerful laser emits a single pulse of uitraviolet radiation lasting $5.0 \times 10^{-9} \mathrm{~s}$. The energy of each photon in the beam is $5.6 \times 10^{-19} \mathrm{~J}$.
(i) Calculate the frequency of an uitra violet photon. the Planck constant $h=6.6 \times 10^{-34} \mathrm{Js}$
frequency $=$ $\qquad$ Hz [2]
(ii) The energy in each pulse is 1.8 MJ .

Show that the pulse contains $3.2 \times 10^{24}$ photons.
(iii) Calculate the power delivered by the laser pulse. Give a suitable unit for your answer.
(b) A photon of the laser light strikes the clean surface of a sheet of metal. This causes an electron to be emitted from the metal surface.

The minimum energy required to release an electron from this surface is $4.8 \times 10^{-19} \mathrm{~J}$.
(l) Show that the maximum kinetic energy of the emitted electron is $8.0 \times 10^{-20} \mathrm{~J}$.
(ii) Show that the speed of an electron with this maximum energy is about $4 \times 10^{5} \mathrm{~ms}^{-1}$.
mass of electron $=9.1 \times 10^{-31} \mathrm{~kg}$
(iii) Electrons are quantum objects. The wavelength $\lambda$ associated with an electron is given by the de Broglie equation

$$
\lambda=\frac{h}{m v}
$$

where $m$ is the mass of the electron and $v$ is the speed at which the electron is travelling.

Calculate the wavelength associated with the emitted electron.

2 Photons from a source reach the point $\mathbf{X}$ on the screen by the two possible paths shown in Fig. 2.1. The resultant phasor amplitude at $\mathbf{X}$ for these two paths is 3.0 .
At another point $\mathbf{Y}$ on the same screen (Fig. 2.2), the resultant phasor amplitude is 1.5.


Fig. 2.1


Fig. 2.2

Calculate the ratio, $P=\frac{\text { probability of photons arriving at point } \mathbf{X}}{\text { probability of photons arriving at point } \mathbf{Y}}$

$$
\begin{equation*}
P= \tag{2}
\end{equation*}
$$

2 A narrow beam of light is always reflected from a mirror at an angle $r$ equal to the angle of incidence $i$, as shown in Fig. 2.1.


Fig. 2.1
Select from the statements (A, B and C) below the one that is the best explanation of this fact, in terms of the quantum behaviour of photons.

A The angles are equal because the photons rebound elastically from the surface.
B Only for paths very close to the observed path do the phasor amplitudes all combine in phase.
C The observed path is the only one along which the momentum of the photon is unchanged.

5 A metal surface is illuminated with light. An electron in the metal surface requires a minimum amount of energy to escape from the surface.

The frequency of the light is changed and the maximum kinetic energy of the electrons emitted is measured for each frequency.

Fig. 5.1 shows a graph drawn using the measurements.


Fig. 5.1
(a) Use the graph to determine the minimum frequency of light required for an electron to escape from the metal surface.
frequency =
$\qquad$ Hz
(b) Explain how the graph suggests that a minimum amount of energy is required for an electron to escape from the metal.
(c) Calculate the gradient of the graph.

## t test : Quantum Physics 7

12 In this question, you are to choose, and write about, a phenomenon in which quantum behaviour is important.
(a) State the quantum phenomenon about which you have chosen to write.
(b) Draw a labelled diagram of the arrangement of apparatus that could be used to observe the quantum phenomenon.
(c) Give a detailed description of what could be observed with this apparatus. Your description may include a diagram.
(d) Use ideas of quantum behaviour to explain the observations described in (c) above. Use equations where appropriate in your explanation.

## t test: Quantum Physics 8

11 This question is about the quantum behaviour of photons.
Yellow light of a single wavelength falls on the vertical surface of a soap film.
Photons of the light reflect from the film and horizontal bands can be seen in the soap film, as shown in Fig. 11.1. The bands are alternately yellow and black.

(a) Fig. 11.2 shows how the percentage of incident photons reflected by the film varies as its thickness changes.

Use the information in Fig. 11.2 to describe in words how the percentage of photons reflected varies with the thickness of the soap film.
(b) An incident photon can reflect off either the front or back surface of the soap film to reach the detector. If it does not reflect, it will pass through the film. (Fig. 11.3)


Fig. 11.3

Some photons reach the detector after reflecting from two different places on the film where the film thickness is $x$ and $y$.

Rotating phasors for the two paths of a photon reaching the detector are shown below, for the two thicknesses of film. (scale: 1 cm represents amplitude 2.0 )
thickness $x$

path 1

path 2
thickness $y$

path 1

path 2
(i) By scale drawing or some other method of your choosing, calculate the magnitude of the resultant phasor amplitude in each case.

Each phasor has an amplitude of 2.0.
thickness $x$

$$
\text { thickness } y
$$

resultant phasor amplitude $=$ $\qquad$
resultant phasor amplitude $=$ $\qquad$
(ii) Show that the probability of photons being reflected from film of thickness $x$ is twice that from film of thickness $y$.
(iii) At certain thicknesses of film, dark bands are produced indicating that few, if any, photons are reflected there.

How do you account for this?

6 This question is about calculating the probability of photons arriving at different points on a screen.


Fig. 6.1
Photons travel from a source $\mathbf{S}$, through three very narrow slits, to fall on a distant screen, as shown in Fig. 6.1.
The diagram shows three possible paths by which a photon could reach the region around $\mathbf{P}$.
The phasors associated with the three paths shown are considered at three different points $\mathbf{P}, \mathbf{Q}$ and $\mathbf{R}$ on the screen. See the table below.

Each phasor is represented by an arrow 1.0 cm long.

| position | phasor diagram | resultant phasor <br> amplitude | relative probability <br> of arrival of photon |
| :---: | :---: | :---: | :---: |
| $\mathbf{P}$ | $\longrightarrow$ | 3.0 |  |
| $\mathbf{Q}$ |  |  | 5.8 |
| $\mathbf{R}$ |  | 1.0 | 1.0 |

(a) Complete the table by filling in each blank space with the missing drawing or number. [3]
(b) Sketch below a phasor diagram to show how it is possible for the three phasors to add to give a relative probability of zero.

5 Photons travel from a distant monochromatic light source to distant detectors $\mathbf{X}$ and $\mathbf{Y}$ through three equally spaced narrow slits, as shown in Fig. 5.1.


Fig. 5.1
(a) At $\mathbf{Y}$, the three phasors associated with the paths from the three slits are


Combine the phasors to show the resultant phasor amplitude for these three paths.
(b) At $\mathbf{X}$, the intensity is maximum.

Calculate the value of the ratio $R=\frac{\text { intensity at } \mathbf{X}}{\text { intensity at } \mathbf{Y}}$.

10 This question is about the quantum behaviour of photons.
(a) Light is emitted from a source A and detected by a detector at B.


Fig. 10.1
Fig. 10.1 shows just three of the many paths a photon might take from $\mathbf{A}$ to $\mathbf{B}$. These paths are very close to a straight line path drawn from A to $\mathbf{B}$.

The path difference between these paths is almost zero, and the phasors associated with these paths will be almost in-phase at B. So, the resultant phasor amplitude at B for these paths is large.
(i) Draw a diagram to show how the phasors at $\mathbf{B}$ for the three paths shown in Fig. 10.1 can combine to give a large resultant phasor amplitude.
(ii) Explain why this resultant phasor amplitude implies a high probability that photons arrive at B along paths like these.
(b) Fig. 10.2 shows three other similarly close, but less direct, paths a photon might take from $\mathbf{A}$ to $\mathbf{B}$.


Fig. 10.2
The path difference between these paths is larger than for the paths shown in Fig. 10.1. This means there is a smaller resultant phasor amplitude at B for these paths, and a lower probability that a photon will take paths like these.
(i) Draw a diagram to show how the phasors at $\mathbf{B}$ for the three photon paths shown in Fig. 10.2 can combine to give a smaller resultant phasor amplitude than in (a)(i).
(ii)

source A


Fig. 10.3
Using a screen to block out most of the photon paths, as shown in Fig. 10.3, makes very little difference to the intensity of light arriving at $\mathbf{B}$.

Use the ideas above to explain why this is so.

3 The table shows the wavelengths and frequencies associated with the photons emitted from three different light emitting diodes (LEDs), labelled A, B and C.

| LED | wavelength <br> $/ 10^{-7} \mathrm{~m}$ | frequency <br> $/ 10^{14} \mathrm{~Hz}$ | colour |
| :---: | :---: | :---: | :---: |
| A | 7.0 | 4.3 |  |
| B | 5.8 | 5.2 |  |
| C | 5.2 | 5.8 |  |

The three colours of light emitted by the LEDs are blue, green and red.
(a) Complete the table by indicating the colour of light emitted by each LED.
(b) Calculate the energy of the photon with greatest energy.
the Planck constant $=6.6 \times 10^{-34} \mathrm{Js}$
energy =

5 A beam of electrons is accelerated through a potential difference $V$ in an electron gun. The beam strikes a graphite target in the evacuated tube and the atoms of the target scatter the electrons.


Fig. 5.1
An intense beam of electrons is detected at an angle $\theta$, as shown in Fig. 5.1.
The angle $\theta$ through which the beam is diffracted is measured for different values of accelerating potential difference $V$. The readings are given below, and values of $\sin \theta$ have been calculated in each case.

| $V / \mathrm{kV}$ | $\theta /$ degrees | $\sin \theta$ |
| :---: | :---: | :---: |
| 3.0 | 12 | 0.208 |
| 4.0 | 10 | 0.174 |
| 5.0 | 9 | 0.156 |

It is suggested that the relationship between the diffraction angle $\theta$ and the accelerating potential difference $V$ is given by:

$$
\sin \theta=\frac{k}{\sqrt{V}} \text { where } k \text { is a constant for the apparatus. }
$$

Propose and carry out a test to check if the relationship is true for these data.

| test proposed | working |
| :--- | :--- |
|  |  |
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## t test : Quantum Physics 12

9 (a) Electromagnetic radiation of wavelength $9.2 \times 10^{-8} \mathrm{~m}$ is incident on a metal surface in a vacuum.
(i) Show that the energy of a photon of this radiation is about $2.2 \times 10^{-18} \mathrm{~J}$.
speed of electromagnetic radiation $=3.0 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$ the Planck constant $h=6.6 \times 10^{-34} \mathrm{~J} \mathrm{~s}$
(ii) When a single photon is absorbed, the energy of the photon can release an electron from the metal. The minimum energy required to release an electron from the metal surface is $4.0 \times 10^{-19} \mathrm{~J}$.

Explain why electrons emerge from the metal with kinetic energies up to a maximum of $1.8 \times 10^{-18} \mathrm{~J}$.
(iii) Show that the speed of an electron of kinetic energy $1.8 \times 10^{-18} \mathrm{~J}$ is $2.0 \times 10^{6} \mathrm{~m} \mathrm{~s}^{-1}$. mass of electron $=9.1 \times 10^{-31} \mathrm{~kg}$
(b) (i) The wavelength $\lambda$ associated with an electron is given by the de Broglie equation

$$
\lambda=\frac{h}{m v}
$$

where $m$ is the mass of the electron and $v$ is the speed at which the electron is travelling.
Calculate the wavelength associated with an electron travelling at $2.0 \times 10^{6} \mathrm{~m} \mathrm{~s}^{-1}$.
(ii) Suggest why a stream of these electrons directed at a thin foil sample in which the interatomic spacing is $4.0 \times 10^{-10} \mathrm{~m}$ could produce a diffraction pattern on the far side of the foil.

6 This question compares wave and photon descriptions of the nature of light. Consider the following situation:

When a laser beam passes through a narrow slit, the beam spreads out and produces broad patches of light on a screen beyond the slit as shown in Fig. 6.1

Fig. 6.1
Complete the table by filling in the missing explanation in each pair.

| wave explanation | quantum explanation |
| :--- | :--- |
| The energy is carried to the screen by <br> electromagnetic waves. | The energy is carried to the screen by <br> photons. |
|  |  |
| The energy arrives continuously at the <br> screen. |  |
| The energy of the electromagnetic wave <br> is proportional to (amplitude) |  |
| wave. |  |

4 Photons of light from a vapour lamp are emitted at only two wavelengths $\lambda_{1}$ and $\lambda_{2}$.
(a) Show that the difference in photon energy $\Delta E$ can be given by the equation

$$
\Delta E=h c\left(\frac{1}{\lambda_{1}}-\frac{1}{\lambda_{2}}\right)
$$

where $h$ is the Planck constant and $c$ is the speed of light in air.
(b) The two wavelengths present in the light are 589.6 nm and 589.0 nm .

Calculate the difference $\Delta E$ in photon energy.

$$
\begin{aligned}
& h=6.6 \times 10^{-34} \mathrm{Js} \\
& c=3.0 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}
\end{aligned}
$$

## t test : Quantum Physics <br> 14

11 In Einstein's explanation of the photoelectric effect, a photon incident on a metal surface must have at least a minimum energy to release an electron from the surface. This minimum energy is called the work function of the metal.

A photocell generates an electric current using the photoelectric effect.
(a) An electron current is generated within the photocell shown in Fig. 11.1 when photons of light of wavelength $5.0 \times 10^{-7} \mathrm{~m}$ are incident on the metal surface of the photocathode.


Fig. 11.1
(i) Show that a single photon of this light has energy of about $4 \times 10^{-19} \mathrm{~J}$.

$$
\begin{aligned}
& h=6.6 \times 10^{-34} \mathrm{Js} \\
& c=3.0 \times 10^{8} \mathrm{~ms}^{-1}
\end{aligned}
$$

(ii) The power incident on the metal surface is 0.1 W .

Show that about $2.5 \times 10^{17}$ photons are incident on the photocathode every second.
(iii) The electron current produced by the photons in the photocell is 1.2 mA .

Show that only $3 \%$ of the photons incident on the photocathode produce electrons which contribute to the current in the circuit.
charge on electron $=1.6 \times 10^{-19} \mathrm{C}$
(iv) Suggest and explain one reason why the number of electrons released is much smaller than the number of photons incident on the metal surface.
(b) The source of light is replaced by a more powerful source of light of wavelength $7.0 \times 10^{-7} \mathrm{~m}$. Even though the power incident on the metal surface from this source is 1.0 W , no current is recorded.

Without calculation, explain why there is no current generated in the photocell.

## t test: Quantum Physics 15

3 The table shows the wavelengths and frequencies associated with the photons emitted from three different light emitting diodes (LEDs) labelled $\mathbf{P}, \mathbf{Q}$ and $\mathbf{R}$.

| LED | wavelength $/ 10^{-7} \mathrm{~m}$ | frequency $/ 10^{14} \mathrm{~Hz}$ | colour |
| :---: | :---: | :---: | :---: |
| $\mathbf{P}$ | 7.0 | 4.3 |  |
| $\mathbf{Q}$ | 5.2 | 5.8 |  |
| $\mathbf{R}$ | 4.6 | 6.5 |  |

The three colours of light emitted by the LEDs are green, red and blue.
(a) Complete the table by indicating the colour of light emitted by each LED.
(b) Calculate the energy of the photon with least energy.
the Planck constant $=6.6 \times 10^{-34} \mathrm{~J} \mathrm{~s}$
energy =

8 Photons travel from a monochromatic light source to detectors $\mathbf{X}$ and $\mathbf{Y}$ through three equally spaced narrow slits, as shown in Fig. 8.1.

photons from distant source


Fig. 8.1
(a) At $\mathbf{X}$, the phasors associated with the paths through the three slits combine to produce a resultant phasor amplitude of 5.0.

State the relationship between resultant phasor amplitude and probability of arrival of photons.
(b) At Y the resultant phasor amplitude is 1.25 .

Calculate the ratio $R=\frac{\text { probability of photons arriving at point } \mathbf{X}}{\text { probability of photons arriving at point } \mathbf{Y}}$.

$$
R=
$$

6 Light from a lamp submerged on the bottom of a pond reaches your eye after passing through the surface of the water. Some possible paths for photons are shown in Fig. 6.1.


Fig. 6.1
Photons could reach your eye by many possible paths. The observed path of a ray of light is the path WXY.
(a) Write down the letter (A, B or C) of the statement which correctly completes the sentence below.

The paths close to the one labelled WXY in Fig. 6.1 are important because ...
A ... the phasors associated with these paths are opposite in phase at $\mathbf{Y}$ to those for all other paths.

B $\quad .$. when the phasors associated with these paths at $\mathbf{Y}$ are placed tip to tail they tend to line up.

C $\quad .$. when the phasors associated with these paths at $\mathbf{Y}$ are placed tip to tail they tend to curl up.
answer
(b) Explain why a photon can take less time to travel from $\mathbf{W}$ to Y by the path $\mathbf{W X Y}$, than by the direct straight path, even though the geometrical distance is greater.

