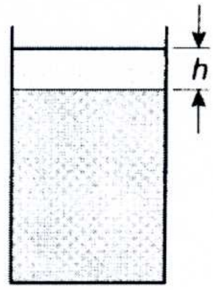
Radioactive Decay Past Paper Questions 2863 and then G494

June 2004

3 It is suggested that the height h of the foam on the top of a fizzy drink (see Fig. 3.1) decreases **exponentially** with time. The table shows data taken to test this assumption.

time / s	0	15	30	45	60
height of foam, h/cm	6.0	4.8	3.8	3.1	2.5
Δh	1·2 a	1·0	0·7	0·6	
$\Delta h / h$	b 0·20	0·21	0·18	0·19	



e.g.
a $6{\cdot}0 - 4{\cdot}8 = 1{\cdot}2$
b $1{\cdot}2/6{\cdot}0 = 0{\cdot}20$

Fig. 3.1

(a) Use this data to test if the height of the foam decreases exponentially with time. State your conclusion.

Since $\Delta h / h \approx$ constant $\Delta h \propto h$ which is condition for exponential decay. → YES it is exponential

[2]

(b) Give another physical example of an exponential decrease.

Radioactive decay

Decay of charge on a capacitor

[1]

11 This question is about the decay of a radioactive substance.

The rate of decay of a radioactive substance is described by the equation

$$\frac{dN}{dt} = -\lambda N$$

where

λ is the probability of any single nucleus decaying in unit time
N is the number of undecayed nuclei present at time t.

(a) Explain how the equation shows that the activity will decrease over time.

The rate of decay is proportional to the number of undecayed nuclei and activity is determined by the rate of decay.

[2]

(b) A sample with 1.0 g of $^{238}_{92}U$ contains 2.5×10^{21} uranium-238 nuclei.

Calculate the activity of the sample.

$\lambda = 5.0 \times 10^{-18}\,s^{-1}$

$A = \lambda N = 5.0\times10^{-18} \times 2.5\times10^{21}$

activity = 12500 s^{-1} [1]

(c) The graph in Fig. 11.1 shows how the natural logarithm of the number (ln N) of uranium-238 nuclei will change with time from an initial sample of 1.0 g of uranium-238.

$\ln 2.5 \times 10^{21} = 49.3$ After 1 half-life number remaining $= 1.25\times10^{21}$

$\ln 2 = 0.693$ $\ln 1.25\times10^{21} = 48.6$

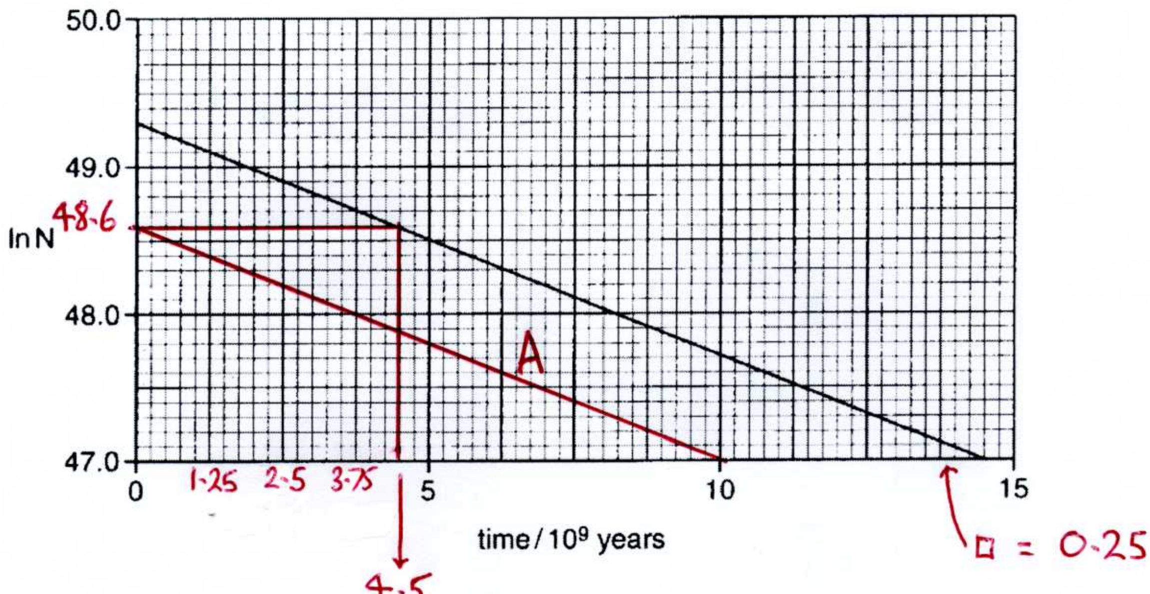


Fig. 11.1

(i) Explain how the graph shows that the half-life is about 4.5 billion years.

ln N = 48·6 after 1 half-life and this maps to a time of 4.5×10^9 years. See graph. [2]

(ii) Explain why the activity of the sample of uranium-238 also falls to half its original value in this period.

As there is half the number of nuclei the number decaying per second (activity) is also half. $A = \lambda N$ [1]

(iii) Draw a line on the graph showing the decay of a 0.5 g sample of uranium-238. Label this line A. → so starts at 48·6 [2]

(d) The stable isotope lead-206 is formed only by the decay of uranium-238. The age of rocks can be estimated by measuring the ratio of lead-206 to uranium-238.

Suggest why rocks are never found with a lead-206 to uranium-238 ratio of more than about 1:1.

age of Earth = 4.6×10^9 years

For a ratio more than 1:1 you need longer than 1 half-life. As the age of the Earth is around a half-life the rocks can never be old enough for this to occur. [1]

[Total: 9]

Jan 2008

3 Radon-220 is a radioactive gas with a half-life of 52 seconds. A sample of the gas is measured to have an activity of 1500 counts s^{-1}.

Calculate the activity 260 seconds later.

$\frac{260}{52} = 5$ half-lives $\quad 1500 \times \left(\frac{1}{2}\right)^5 =$

activity = ...47.... counts s^{-1} [2]

or $\lambda = \frac{\ln 2}{52} = 0{\cdot}01333\ s^{-1}$

$A = A_0 e^{-\lambda t} = 1500 e^{-0.01333 \times 260} = 47$

5 A student produces a simple model of radioactive decay using the following equation

rate of decay of sample $\dfrac{\Delta N}{\Delta t} = -\lambda N$

where N is the number of nuclei present
λ is the decay constant
Δt is a small interval of time
ΔN is the number of nuclei decaying in time Δt.

The student chooses to set λ at $0.14\,s^{-1}$, the initial number of nuclei at 9.0×10^5 and the time interval Δt between calculations at 1.0 s. It is assumed that the rate of decay is constant over each time interval.

(a) Show that according to this model the number of nuclei remaining after 1.0 s is about 7.7×10^5.

$\Delta N = 9.0 \times 10^5 \times 0.14 = 1.26 \times 10^5$

$N = N_0 - \Delta N = 9.0 \times 10^5 - 1.26 \times 10^5 = \underline{7.74 \times 10^5}$

[2]

(b) The student uses the model to calculate the number of nuclei remaining at successive one second time intervals for a period of 8 seconds. These results are shown in Fig. 5.1.

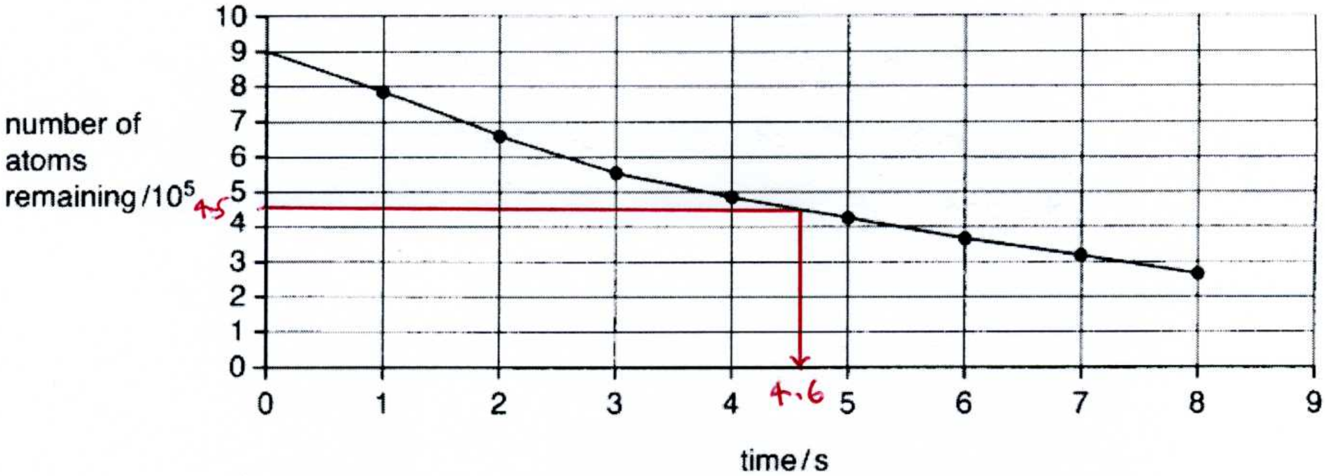


Fig. 5.1

(i) Show on the graph that the model gives a half-life of about 4.6 s. [1]

(ii) The actual half-life of an isotope with a decay constant of $0.14\,s^{-1}$ is 5.0 s. Account for the inaccuracy of the model and suggest how the model could be improved to give a closer match to reality.

The model keeps the rate of decay constant during each 1s time interval - it actually falls though. To improve the model make Δt smaller.

[2]

4 Protoactinium decays with a decay constant λ of $9.7 \times 10^{-3}\,s^{-1}$.

(a) Show that the expected number of decays in one second in a sample of 10^4 protoactinium atoms is about $10^2\,s^{-1}$.

$A = \lambda N = 9.7 \times 10^{-3} \times 10^4 = 97$

[1]

(b) A student suggests that all the protoactinium will have decayed after about 100 s and draws the graph in Fig. 4.1 to explain why.

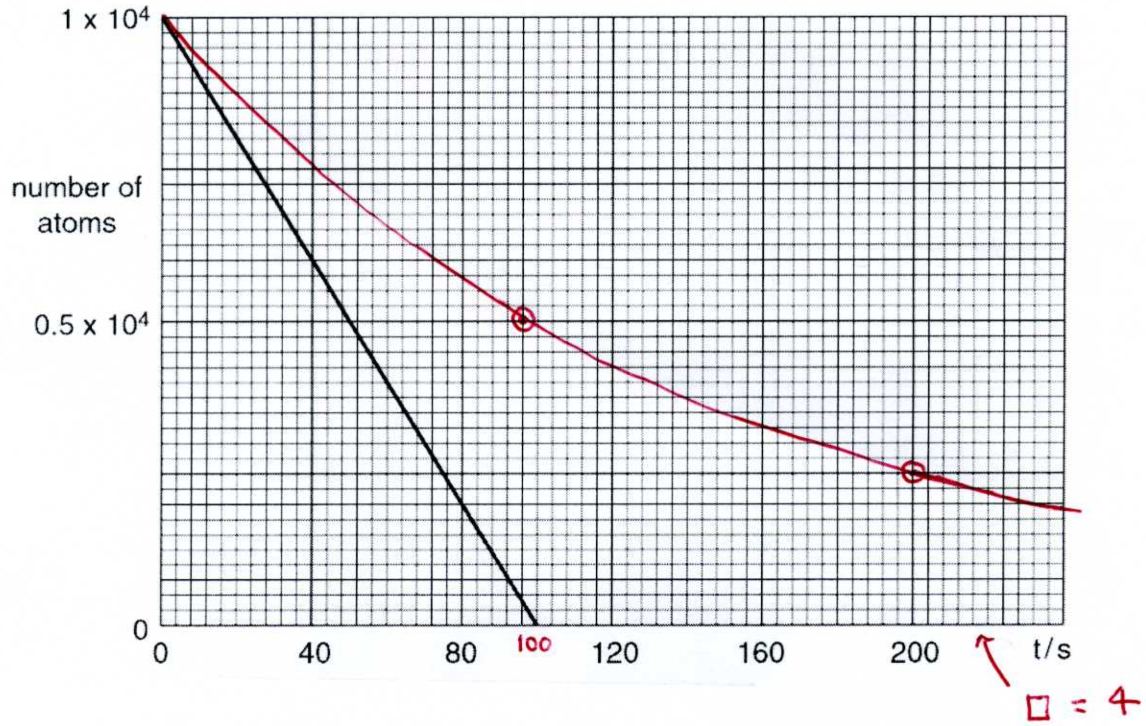


Fig. 4.1

(i) Sketch a more appropriate decay graph on Fig. 4.1. [2]

(ii) Explain why the student's reasoning is wrong.

The rate of decay is not constant it is propotional to the number of nuclei remaining.

[1]

8 This question is about an americium-241 radionuclide source used in a smoke detector.

An image had been removed due to third party copyright restrictions

Details: An image of a smoke detector

Fig. 8.1

Here are some data about the source:

activity of source = 3.3×10^{4} Bq

decay constant, λ = 4.8×10^{-11} s^{-1}

(a) Show that the source initially contains about 7×10^{14} americium nuclei.

$$A = \lambda N \quad \therefore \quad N = A/\lambda = \frac{3.3 \times 10^{4}}{4.8 \times 10^{-11}}$$

$$= \underline{6.9 \times 10^{14}}$$

[2]

(b) Show that the half-life of the source is about 450 years.

1 year = 3.2×10^{7} s

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{4.8 \times 10^{-11}} = 1.44 \times 10^{10} \text{ s}$$

$$\frac{1.44 \times 10^{10}}{3.2 \times 10^{7}} = \underline{451 \text{ years}}$$

[3]

6

(c) The smoke detector works by detecting a decrease in the arrival of alpha particles from the americium when they are absorbed by the smoke. It is advised that the smoke detector is replaced after five years.

Use the equation $\Delta N = -\lambda N \Delta t$ with N taken as the original number of nuclei to show that about 5×10^{12} nuclei decay in the five years of use.

$$\Delta N = -\lambda N \Delta t \quad = \quad -4.8 \times 10 \times \underbrace{6.9 \times 10^{14}}_{\text{from (a)}} \times \underbrace{5 \times 3.2 \times 10^{7}}_{t \text{ in years.}}$$

$$= -5.3 \times 10^{12}$$

[2]

(d) Explain why

- it is reasonable to use the equation $\Delta N = -\lambda N \Delta t$ (as above) to estimate the number of nuclei decaying over a five year period
- the equation used in this way would **not** give an accurate answer for the number of nuclei decaying over a few hundred years.

Over 5 years N does not change much so reasonable to assume rate is constant. Over a few hundred years N will change ($t_{1/2} = 450$ yrs) so rate will not be constant (rate $\propto$ N)

[2]

(e) Explain why a decrease in activity of the sample is not likely to be the reason that the smoke detector should be replaced after five years. Suggest a more likely reason that the detector might need replacing.

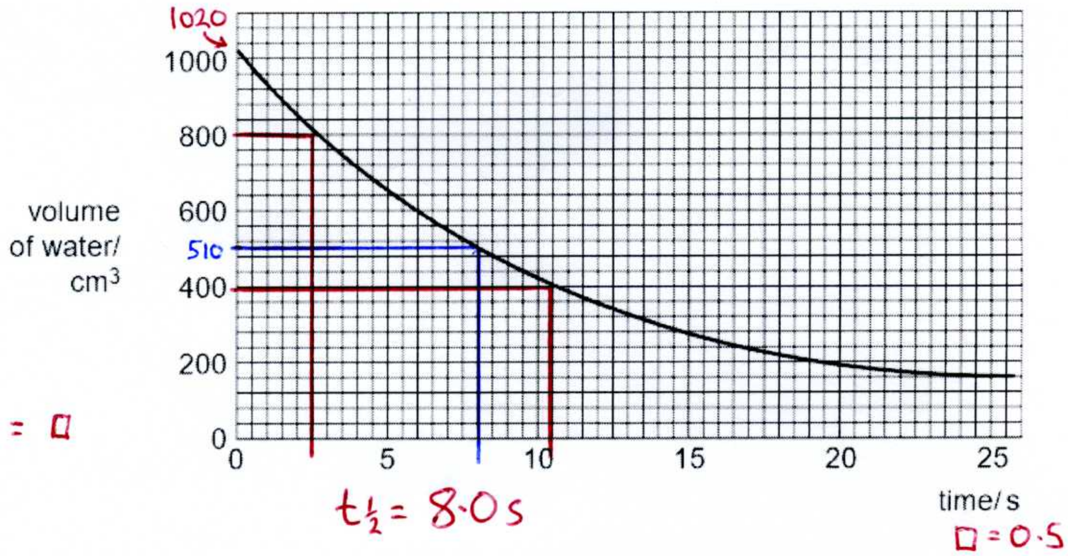
Very little change in activity after 5 years.

Reason could be a build up of dust or life-time of electrical components / battery.

[2]

[Total: 11]

6 An experiment is performed to measure the volume of water in a can at regular intervals as the water drains through a small hole. A graph of the results is shown in Fig. 6.1.



40 = $\square$

Fig. 6.1

The volume of water in the can is changing exponentially. The rate of change of volume, $\frac{\Delta V}{\Delta t}$, is given by the equation

$$\frac{\Delta V}{\Delta t} = -\phi V$$

where ϕ is a constant.

(a) Suggest one way to modify the can, hole or liquid to increase the value of ϕ.

larger hole / lower viscosity / hole lower in can etc

[1]

(b) ϕ can by found from the graph using the relationship

$$\phi = \frac{\ln 2}{t_{1/2}}$$

where $t_{1/2}$ is the time for the volume of water in the can to reduce to half its original value. Use the graph to estimate a value for ϕ.

$$\phi = \frac{\ln 2}{8} =$$

ϕ = 0·087 unit s^{-1} [3]

10 This question is about how radioactive carbon-14 has been used to estimate the date of construction of an ancient stone circle.

(a) Living matter has 4.0×10^{10} atoms of carbon-14 in every gram of carbon giving an activity of about 0.16 counts s^{-1}.

(i) Use this information to calculate the decay constant λ of carbon-14.

$A = \lambda N \quad \therefore \quad \lambda = A/N = 0.16/4\times10^{10} =$

$\lambda =$ 4×10^{-12} s^{-1} **[2]**

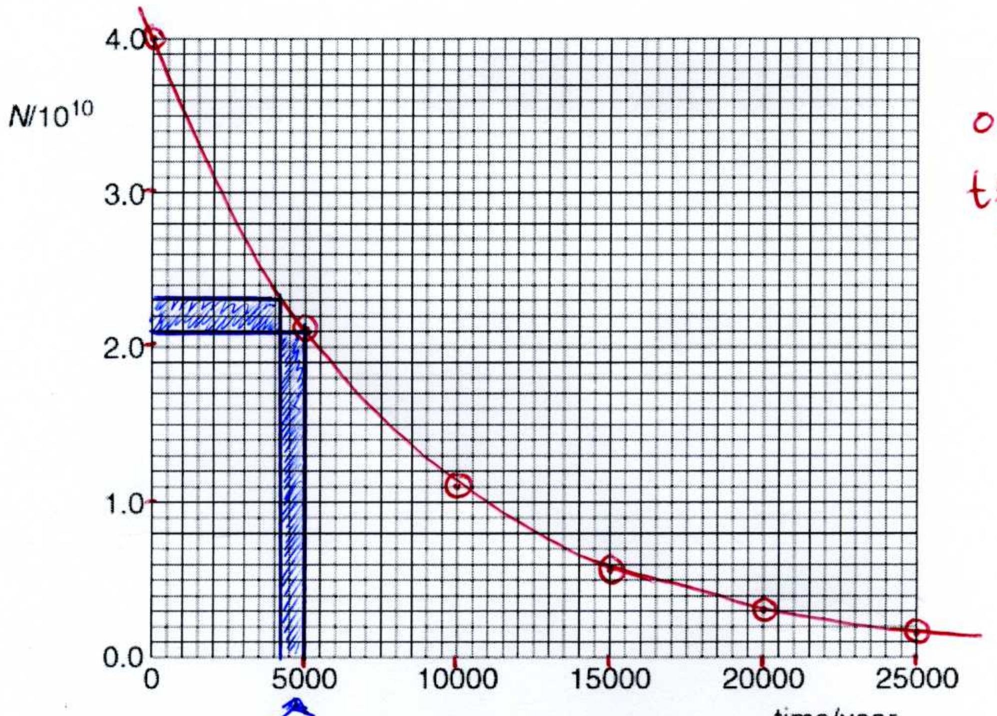
(ii) Show that the half-life of carbon-14 is about 5500 years.
1 year $= 3.2 \times 10^{7}$ s

$t_{\frac{1}{2}} = \ln2/\lambda = \ln2/4\times10^{-12} = 1.73\times10^{11}$ s

$1.73\times10^{11}/3.2\times10^{7} = 5400$ years **[2]**

(b) (i) Draw a graph on Fig. 10.1 to show how the number N of carbon-14 atoms per gram of carbon varies over time. **[2]**

Use $N = N_0 e^{-\lambda t}$ in calc & update t



or use $t_{\frac{1}{2}} = 5400$ yrs.

N= 2.1 1.1 0.59 0.31 0.16

Fig. 10.1

4500 ± 500 yrs

(ii) A fragment of deer horn was found at the base of one of the large stones. It is assumed that the horn was buried when the stone was laid in place and that the horn had 4.0×10^{10} atoms of carbon-14 per gram of carbon at the time of burial.

The number of carbon-14 atoms per gram of carbon in samples from the horn varied between 2.1×10^{10} and 2.3×10^{10}.

Use the graph to estimate the age of the sample. State the uncertainty in your estimate.

↗ 250 – 500 OK.

age = 4500 uncertainty ± 500 years **[2]**

(c) Carbon dating is used to date samples of organic material of up to 50,000 years old.

(i) State how many half-lives of carbon-14 represent a time interval of 50,000 years.

$50{,}000 / 5400 = 9.3$

[1]

(ii) Calculate the activity of one gram of carbon in a sample of organic matter remaining after 50,000 years.

Assume the original activity per gram was 0.16 counts s^{-1}.

$0.16 / 2^{9.3}$

activity per gram = 2.5×10^{-4} counts s^{-1} **[1]**

(iii) Suggest why this method of carbon dating is not suitable for determining the age of objects older than 50,000 years.

The count rate is too low to measure (probably less than background)

[1]

(d) It is very important to make sure that the ancient organic matter is not contaminated with modern organic matter.

Explain how contamination with modern matter can affect the calculated age of the sample and suggest whether younger or older samples will be more sensitive to the effects of contamination.

Modern organic matter still contains lots of undecayed ^{14}C so samples will appear younger. The oldest samples will be most affected. (as they have little of their own ^{14}C.)

[3]

[Total: 14]

June 2009

7 A sample of uranium-238 contains 6 x 10^{19} nuclei. The sample has an activity of 300 decays per second. Calculate the decay constant λ of the uranium isotope.

$$A = \lambda N \quad \therefore \quad \lambda = A/N = 300/6\times10^{19} =$$

decay constant = 5.0×10^{-18} s^{-1} **[2]**

Jan 2010

10 This question is about using a radioisotope of potassium to find the age of a rock.

(a) A sample of potassium-40 has a mass of 2.3×10^{-6} g.

(i) Calculate the number of potassium nuclei in the sample.

molar mass of potassium-40 = 40 g mol^{-1}
$N_A = 6.0\times10^{23}\,mol^{-1}$

$$\frac{2.3\times10^{-6}}{40}\times 6.0\times10^{23} =$$

number of nuclei in sample = 3.45×10^{16} **[2]**

(ii) The activity of the sample is 0.57 Bq. Calculate the decay constant λ for potassium-40.

$$A = \lambda N \quad \therefore \quad \lambda = A/N = 0.57/3.45\times10^{16}$$

decay constant = 1.65×10^{-17} s^{-1} **[2]**

(iii) Use the value of λ to calculate the half-life of potassium-40 in years.

1 year = 3.2×10^{7} s

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{1.65\times10^{-17}} = 4.20\times10^{16}\ s$$

$$\frac{4.2\times10^{16}}{3.2\times10^{7}} =$$

half-life = 1.31×10^{9} years **[2]**

(b) Potassium-40 decays into a stable isotope of argon.

A particular rock sample was found to contain numbers of potassium and argon nuclei in a ratio of one nucleus of potassium to three nuclei of argon. 1K : 3Ar

It is assumed that all the argon in the rock has been produced from the decay of potassium, and that none has escaped.

(i) Estimate the number of half lives passed since the rock was formed and calculate the age of the rock.

$t_{\frac{1}{2}}$	0	1	2	3
Ar	0	0·5	0·75	0·875
K	1	0·5	0·25	0·125
K:Ar	1:0	1:1	1:3	1:7

↳ so 2 half-lives

age of rock = 2.6×10^{9} years **[2]**

(ii) In fact, some argon does escape from the rock. Explain the effect this would have on the calculated age of the rock.

The rock would appear to be younger than it actually is as the K:Ar ratio would have more K proportionally so it will look like it has not had as long to decay.

[3]

[Total: 11]

7 A sample of 1.0 g of uranium-238 contains 2.5×10^{21} nuclei. The graph in Fig. 7.1 shows how the natural logarithm ln N of the number N of uranium-238 nuclei changes with time.

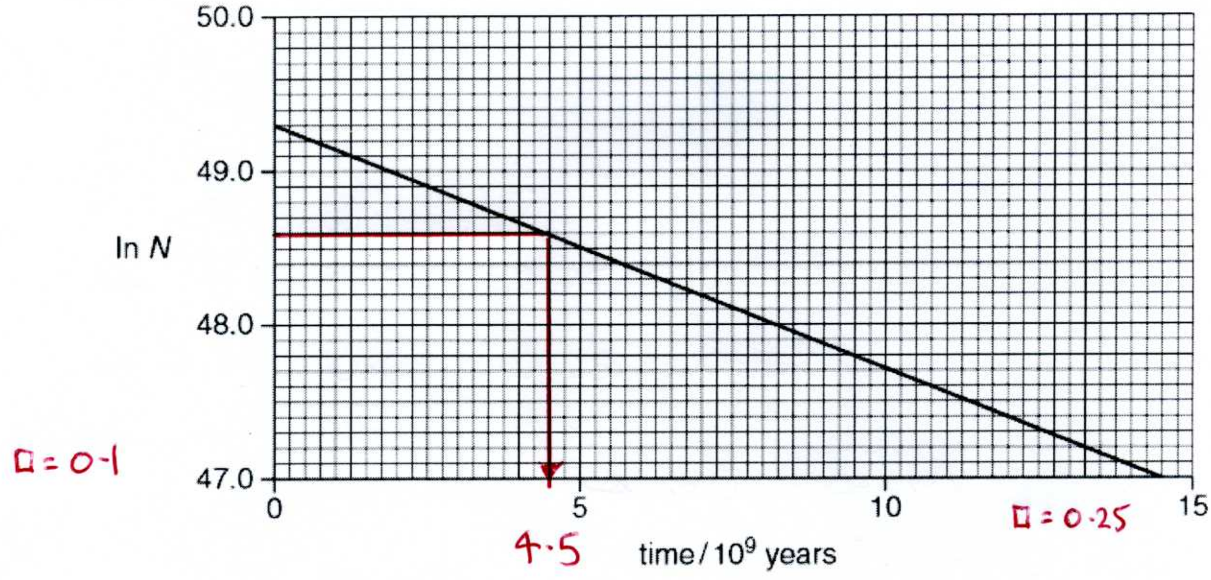


Fig. 7.1

(a) Explain what is meant by the half-life of uranium-238.

The time for half the U-238 nuclei in a sample to decay.

[1]

(b) Use information from the graph to calculate the half-life of uranium-238.

$\ln 2.5 \times 10^{21} = 49.3$

$$\ln\left(\frac{2.5\times10^{21}}{2}\right) = 48.58$$

$\rightarrow$ 4.5×10^9 years

half life = 4.5×10^9 years **[2]**

Jan 2010 G494

2 Sodium-24 is a radioisotope which decays with a half-life of 15 hours.

(a) Calculate a value for the decay constant λ of sodium-24.

$1\text{ h} = 3.6 \times 10^3\text{ s}$

$$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}} = \frac{\ln 2}{15 \times 3.6 \times 10^3} =$$

λ = 1.28×10^{-5} s^{-1} **[1]**

(b) The decay of sodium-24 can be modelled as a random phenomenon.

What is the significance of the decay constant in this model?

probability of decay of a single nucleus per second

[2]

June 2010 G494

4 Strontium-90 is widely used as a source of beta particles.
The activity of a strontium-90 source is 1.6×10^5 Bq.

(a) Calculate the number of strontium-90 nuclei in the source.
The decay constant of strontium-90 is $7.6 \times 10^{-10}\text{ s}^{-1}$.

$$A = \lambda N$$

$$N = \frac{A}{\lambda} = \frac{1.6 \times 10^5}{7.6 \times 10^{-10}}$$

number of nuclei = 2.1×10^{14} **[1]**

(b) Calculate the activity of the source in fifty years time.

$1\text{ year} = 3.2 \times 10^7\text{ s}$

$$A = A_0 e^{-\lambda t} = 1.6 \times 10^5 \times e^{-7.6 \times 10^{-10} \times 50 \times 3.2 \times 10^7}$$

activity = 4.7×10^4 Bq **[1]**

13 This question is about time dilation for particles called muons moving at high speed.

(a) Muons are short-lived particles which are created when protons collide with nuclei at high energy. They decay randomly into electrons and anti-neutrinos, with a half-life of 1.5 μs. The process can be modelled with the expression

$$\frac{\Delta N}{\Delta t} = -\lambda N.$$

(i) Explain the meaning of the decay constant λ in the expression.

Probability of decay of a muon in 1 second.

[2]

(ii) Calculate the value of λ for the decay of a muon.

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{1.5 \times 10^{-6}}$$

λ = 4.62×10^{5} s^{-1} **[1]**

(b) In a recent experiment, a beam of high-energy muons was created with a speed of almost $3.0 \times 10^{8}\,\mathrm{m\,s^{-1}}$. They were trapped in a magnetic field so that they travelled in a circular path until they decayed into electrons. Non-relativistic calculations were made to estimate the time and distance for the muons to decay.

(i) Show that when only one-eighth of the original number of muons remain in the beam, they have travelled about 1.4 km.

The half-life of a muon is 1.5 μs.

$$1 \rightarrow \frac{1}{2} \rightarrow \frac{1}{4} \rightarrow \frac{1}{8}$$

For $\frac{1}{8}$ to remain 3 half-lives must pass

$= 3 \times 1.5 \times 10^{-6}\,s = 4.5 \times 10^{-6}\,s.$

$s = vt = 3 \times 10^{8} \times 4.5 \times 10^{-6} = \underline{1.35\ km}$

[3]

(ii) On the axes of Fig. 13.1, sketch a graph to show how the proportion of muons still in the beam should vary with the distance that they have travelled, assuming the non-relativistic calculation of **(i)**.

Distance per half-life

$= 1.5 \times 10^{-6} \times 3 \times 10^{8}$

$= 0.45$ km

= half-distance

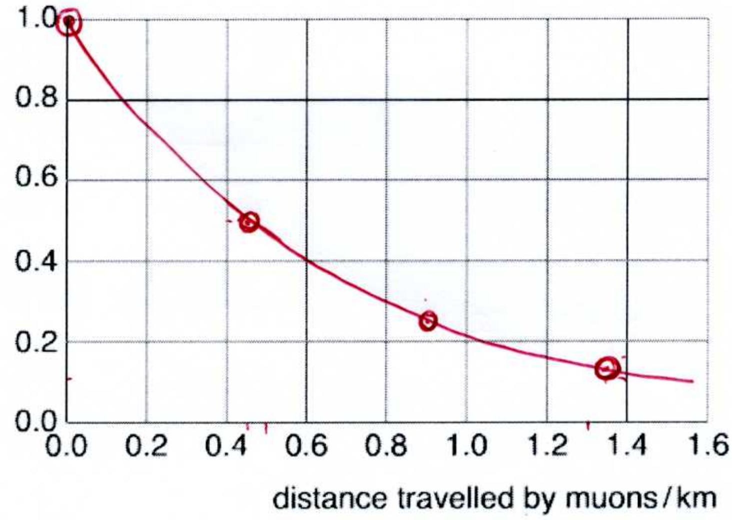


Fig. 13.1

[2]

(iii) The experiment shows that the non-relativistic calculation of **(i)** is wrong.

The muons in the beam are able to travel a distance of 4.0 km before only one-eighth of them are left undecayed. Use the formula

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

to help explain why this is different from your answer to **(i)**.

Your answer should include a value for γ.

$1.35 \times \gamma = 4.0$ km

$\therefore \gamma = 4.0/1.35 = 2.96$ which is the time dilation factor. The half-life will now be $1.5 \times 2.96 = 4.4\,\mu s$ and

[3]

[Total: 11]

$v = 2.8 \times 10^{8}\ ms^{-1}$ as $v = \left(\sqrt{1-\frac{1}{\gamma^2}}\right) \times c$