Simple Harmonic Motion (2863 Jan 04 to Jan 08) Name SA

2863 Jan 04

7 The variation in depth of water in a harbour can be modelled as a simple harmonic oscillation.

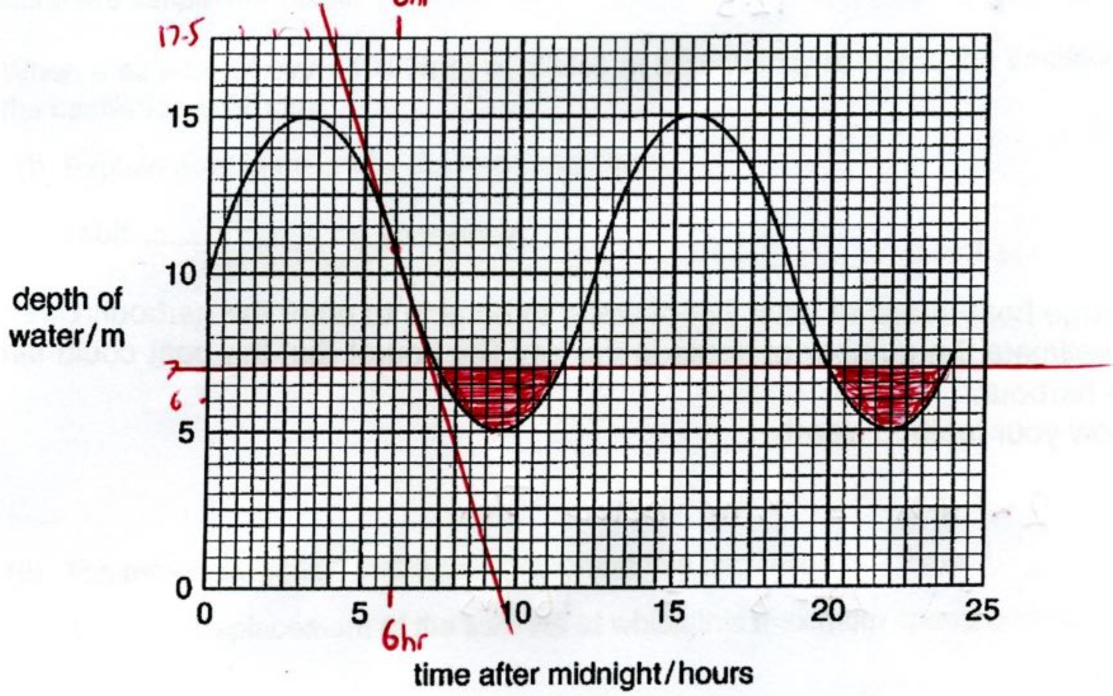


Fig. 7.1

Fig. 7.1 shows a graph produced from the model. It shows the variation of depth of water in a harbour with time over the period of one day (24 hours).

(a) Use the graph to find

(i) the maximum depth of the water in the harbour

maximum depth = ...15.... m [1]

(ii) the amplitude A of the tidal motion

amplitude of motion = ...5.... m [1]

(iii) the rate of change of depth in metres per hour (m hr^{-1}) at $t = 6$ hours where t = time after midnight in hours. Show your working.

$$v = \frac{-17.5}{9.5-3.75} = \underline{-3.0 \text{ m}hr^{-1}}$$

OR

$$v = \frac{dd}{dt} = 2\pi f A \cos 2\pi f t$$

$$f = 1/12.5 = 0.08$$

$$v = 2\pi \times 0.08 \times 5 \cos 2\pi \times 0.08 \times 6$$

$$= \underline{-2.5 \text{ m}hr^{-1}}$$

rate of change of depth = ... m hr^{-1} [3]

(b) Use data from the graph to calculate the frequency f of the tidal motion in units of tides per hour.

$$f = \frac{1}{T} = \frac{1}{12.5} = 0.08\ \text{hr}^{-1}$$

frequency = ... tides per hour [3]

(c) A large boat needs at least 7 m of water to be able to enter the harbour. Use the graph to estimate the number of hours in the day (24 hours) that the boat could safely enter the harbour.
Show your method clearly.

$2 \times 4h = 8$ hr below 7m

$\therefore 24 - 8 = 16$ hrs

[2]

(d) The equation for the depth of water d, in metres, in the harbour is

$$d = 10 + A \sin (2\pi f t).$$

Use your answer to **(a)(ii)** and this equation to show that the lowest depth of water is 5 m.

At lowest point $t = 9.5$ or 22 hrs

$d = 10 + 5 \sin (2\pi \times 0.08 \times 22)$

$= 5.01$ m

(at 9.5h $d = 5.01$ m)

[2]

[Total: 12]

June 2004

8 This question is about the vibrational testing of a satellite.

The vibrations simulate the shaking the satellite would undergo during launch. Assume the vibrations are simple harmonic.

(a) When a satellite of mass 10 000 kg oscillates at a frequency of 10 Hz, the amplitude of the oscillation is 35 mm.

(i) Explain what is meant by the term *amplitude*.

Max displacement from undisturbed position or equlibrium position

[2]

(ii) The maximum speed of the satellite during this oscillation is 2.2 m s^{-1}.

State the displacement of the satellite at which this maximum speed occurs.

displacement ....0......... mm [1]

(iii) State the acceleration of the satellite at the instant that maximum speed occurs.

acceleration ....0......... m s^{-2} [1]

(iv) Calculate the **maximum** acceleration of a satellite due to this oscillation.

$a = -\omega^2 x = -(2\pi f)^2 x$

$= 4\pi^2 \times 10^2 \times 35 \times 10^{-3} = 138 \text{ ms}^{-1}$

maximum acceleration .... m s^{-2} [2]

(v) Show that the satellite will experience accelerating forces far greater than its weight.

$g = 9.8\,\text{N kg}^{-1}$

W = mg = 10,000 × 9·8 = 98000 N

F = ma = 10,000 × 138 = 1380000 N

or 138/9.8 = 14x greater.

[2]

(b) The communications aerial on the satellite is observed to shake violently at one particular frequency. This is an example of resonance.

(i) Explain why the aerial behaves in this way.

Driving frequency = natural frequency

[2]

(ii) Suggest and explain how the aerial could be modified in order to reduce the problem of resonance.

(stick mass on end to lower f
make aerial stiffer to increase f)

Increase damping with some sort of shock absorber.

[2]

[Total: 12]

Taper aerial so resonant f is not same down length of aerial.

Jan 2005

11 This question is about some of the physics of the human ear.

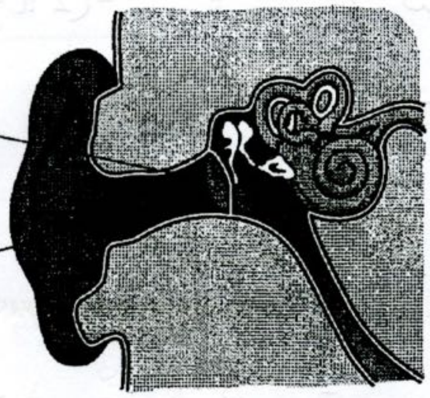


Fig. 11.1

A given sound wave striking the ear drum sets it oscillating in simple harmonic motion.

The ear drum oscillates at a frequency of 2500 Hz with an amplitude of 1.0×10^{-7} m. $= 0.1\,\mu m$

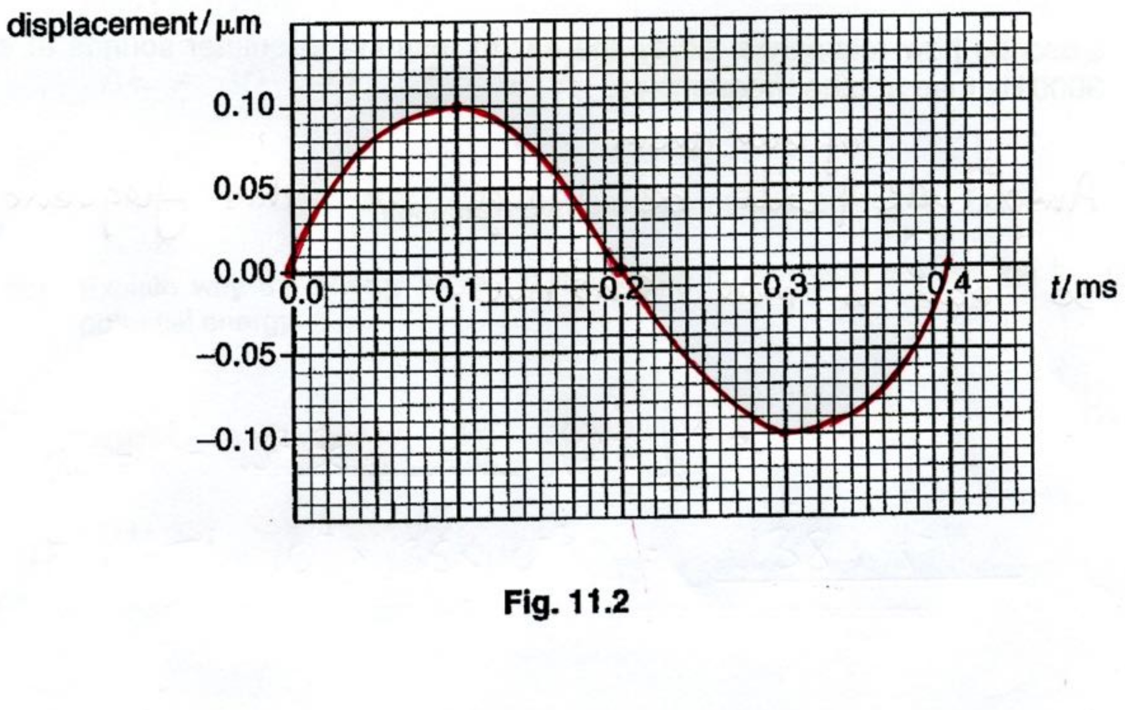
(a) (i) Calculate the period of the oscillation.

$T = 1/f = 1/2500 =$

period = 4×10^{-4} s [1]

$= 0.4$ ms

(ii) On the axes of Fig. 11.2, draw a graph to show how the displacement of the eardrum varies with time for **one** oscillation. Assume that the displacement is zero at $t = 0$.



[3]

Fig. 11.2

(iii) Calculate the maximum acceleration of the ear drum.

$a = -\omega^2 x = -(2\pi f)^2 x$

$= 4\pi^2 \, 2500^2 \times 1.0 \times 10^{-7} =$

acceleration = 24.7 m s^{-2} [2]

(b) At a frequency of 3000 Hz, the ear can detect pressure changes as small as 4.0×10^{-5} N m^{-2}.
Calculate the change of force such a pressure change would give on an ear drum of area 20 mm^2.

$P = F/A \therefore F = P \times A = 4 \times 10^{-5} \times 20 \times 10^{-6}$

$= 8 \times 10^{-10}$ N

change of force = ... N [2]

(c) The human ear is most sensitive to frequencies around 3000 Hz. This is because air in the auditory canal resonates at this frequency.

(i) Explain what is meant by *resonance*.

Driving frequency = natural frequency which leads to large amplitude oscillations

[1]

(ii) Describe how resonance allows the ear to respond to quieter sounds at around 3000 Hz than at other frequencies.

Amplitude of ear drum will be large at this frequency so ear is more sensitive.

[2]

[Total: 11]

Jan 2006

10 This question is about a mass oscillating on an elastic spring.
The spring constant k for the spring is $24\ \mathrm{N\,m^{-1}}$.

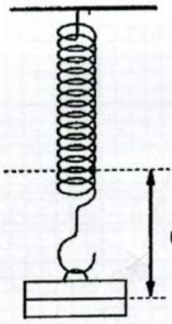


Fig. 10.1

(a) A mass of 0.20 kg is hung from the end of the spring and the system comes to rest.

(i) Show that the extension of the spring is about 0.08 m.

$g = 9.8\ \mathrm{N\,kg^{-1}}$

$F = kx \quad \therefore \quad x = F/k = \dfrac{0.20 \times 9.8}{24} = 0.082\ \mathrm{m}$

[1]

(ii) Calculate the elastic strain energy stored in the spring.

$E = \frac{1}{2}kx^2 = \frac{1}{2} \times 24 \times 0.08^2 = 0.077\ \mathrm{J}$

elastic strain energy = ...J [2]

(iii) Calculate the change in gravitational potential energy of the mass as it extends the spring by 0.08 m.

$\Delta E = mg\Delta h = 0.2 \times 9.8 \times 0.08 = 0.157\ \mathrm{J}$

change in gravitational potential energy = ...J [2]

(iv) Explain why the energy stored in the spring is less than the change in gravitational potential energy.

Some energy is lost as heat as the spring extends.

[1]

(b) The spring is extended by a further 0.060 m and then released. The mass oscillates up and down.

The motion of the mass is recorded using a data logger. A short section of the trace is shown in Fig. 10.2.

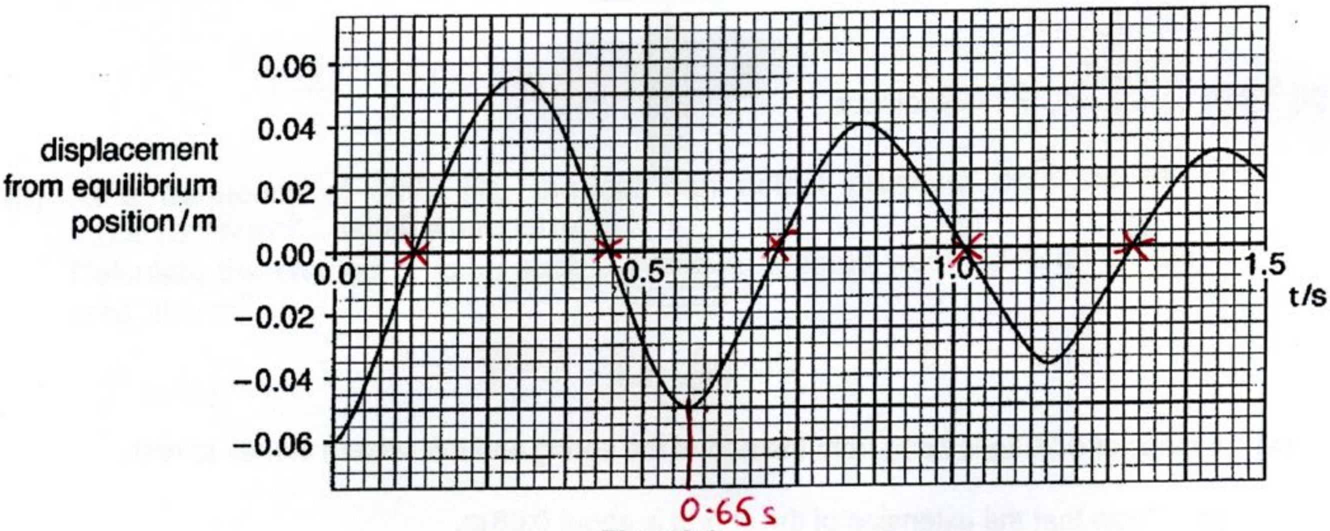


Fig. 10.2

(i) State how the graph shows that the velocity of the oscillating mass is zero when it is at maximum displacement.

The gradient of the line = 0 ms^{-1} at max displacement

[1]

(ii) Mark a point on the graph where the resultant force on the oscillating mass is zero. Label this point **X**. Any point where disp = 0 m [1]

(c) The time trace of the oscillator over a large number of oscillations is shown in Fig. 10.3.

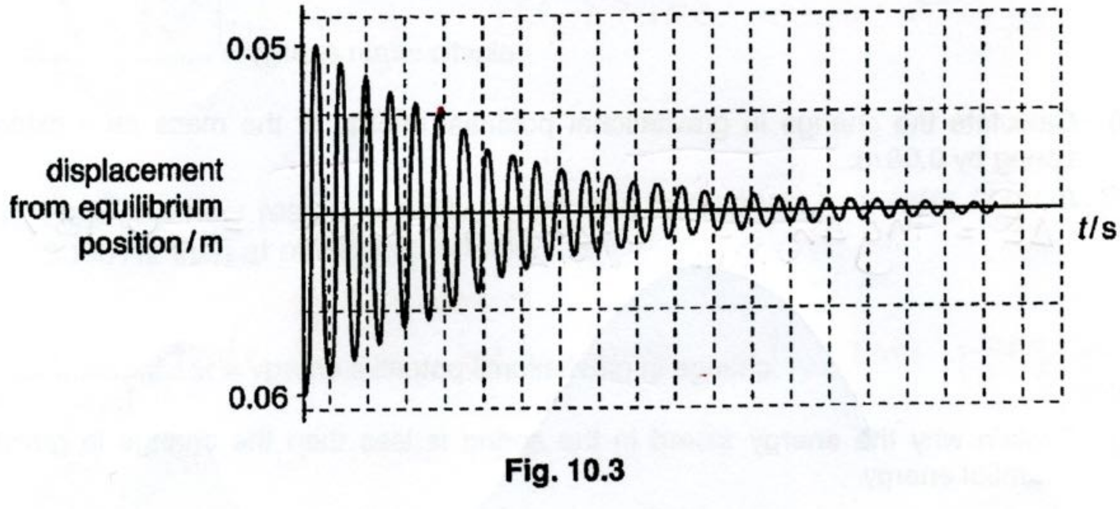


Fig. 10.3

Explain how the graph shows that the amplitude of the oscillation falls exponentially over time.

Amplitude falls by constant factor in equal time intervals.

[2]

[Total: 10]

$\left(t_{\frac{1}{2}} \approx 5\frac{1}{2} \text{ oscillations} \approx 5.5 \times 0.575 = 3.2\,s\right)$

Jan 2007

9 This question is about the oscillation of a mass between a pair of springs as shown in Fig. 9.1.

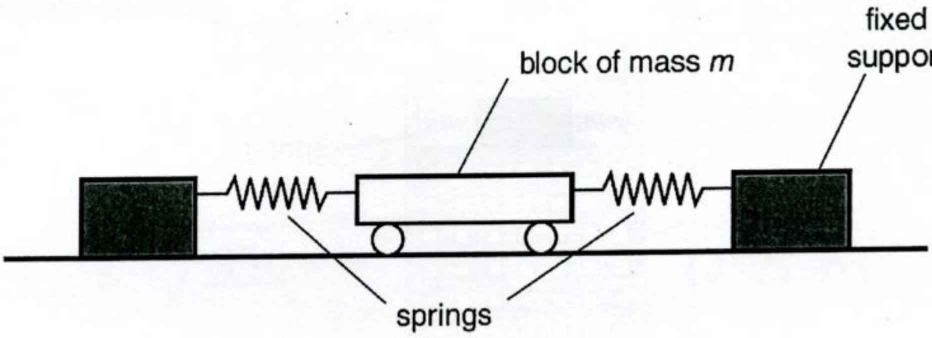


Fig. 9.1

(a) The system obeys Hooke's Law with a stiffness constant k.

The block is displaced a horizontal distance x and released.

(i) Show that the initial acceleration a of the mass m is given by

$$a = -\frac{kx}{m}.$$

$F = -kx$

$F = ma \quad \therefore a = \frac{F}{m} = \frac{-kx}{m}$

[2]

(ii) Explain why the equation in **(i)** shows that the body will undergo simple harmonic motion.

$a \propto -x$ since m is constant.

This is the requirement for SHM.

[2]

(iii) The acceleration of the mass is also given by the equation

$$a = -4\pi^2 f^2 x.$$

Use this equation and the equation from **(i)** to show that

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

$a = \frac{-kx}{m} = -4\pi^2 f^2 x$

[1]

$\therefore \frac{k}{m} = 4\pi^2 f^2 \quad \therefore f^2 = \frac{1}{4\pi^2}\frac{k}{m}$

$\therefore f = \sqrt{\frac{1}{4\pi^2}\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(b) Such a system is used as a **damper** to reduce the movement of tall buildings in earthquakes or high winds as shown in Fig. 9.2.

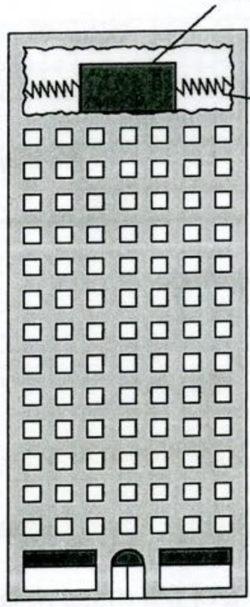


Fig. 9.2 **not to scale**

Here are some data for this damper system.

mass of block = 290 000 kg
stiffness constant of the system = $2.8 \times 10^6\,\mathrm{N\,m^{-1}}$

The system is designed to reduce the oscillations of a building which has a natural frequency of 0.5 Hz.

(i) Show that the frequency of the mass oscillating between springs matches the natural frequency of the building.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2.8\times10^6}{290\,000}} = 0.49\ \mathrm{Hz}$$

[2]

(ii) A sudden movement of the building displaces the block 0.7 m from its equilibrium position relative to the building.

Show that the energy transferred to the oscillator is about 700 kJ.

$$E = \frac{1}{2}kx^2 = \frac{1}{2}\times 2.8\times10^6 \times 0.7^2$$

$$= 6.86\times10^5\ \mathrm{J} \approx 700\ \mathrm{kJ}$$ [1]

(c) The oscillator is damped. It loses 50% of its **energy** on each oscillation.

(i) Show that the **amplitude** of the oscillator is reduced from 0.7 m to 0.5 m after one complete oscillation.

After one oscillation $E = 700/2 = 350\ \text{kJ}$

$$350 \times 10^3 = \frac{1}{2} \times 2.8 \times 10^6\ x^2$$

$$\therefore x = \sqrt{\frac{350 \times 10^3 \times 2}{2.8 \times 10^6}} = 0.5\ \text{m}$$

[2]

(ii) Calculate the number of oscillations it takes for the amplitude to fall to one eighth of its original value.

$1/8$ amplitude $= 1/64$ energy

$= 2^N = 64 \quad \therefore N = 6$

$\frac{0.5}{0.7} = 0.71 \qquad 0.71^N = 0.125 \quad N \approx 6$

number of oscillations = ...6........ [2]

(d) Suggest why dampers are more effective when placed near the top of the buildings.

Amplitude of buildings oscillation is greatest at top so more energy absorbed.

[1]

[Total: 13]

June 2007

10 This question is about investigating the physics of bungee jumping.

A student models a bungee jump by hanging a tennis ball of mass 0.12 kg from a single strand of elastic thread with unstretched length of 0.95 m as shown in Fig. 10.1.

A diagram has been removed due to third party copyright restrictions

Details: A diagram of a tennis ball attached to an eleastic thread which is attached to a clamp stand

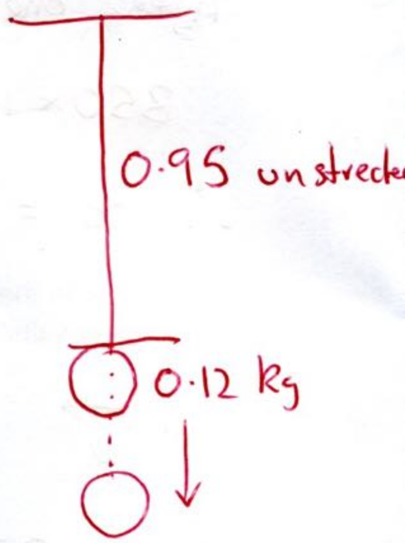


Fig. 10.1

(a) (i) Show that the weight of the ball is about 1.2 N.

$g = 9.8\ \text{N kg}^{-1}$

$W = mg = 0.12 \times 9.8 = 1.18\ N$

[1]

(ii) The elastic thread has a stiffness constant of 3.1 N m $^{-1}$. Show that the elastic thread will stretch to a length of about 1.3 m when the ball is hanging from the end.

$F = -kx \quad \therefore \quad x = F/k = 1.2/3.1 = 0.39\ m$

$0.95 + 0.39 = 1.34\ m$

[2]

(iii) Explain why the resultant force on the tennis ball is zero when the thread is at this length.

Tension in elastic = - weight of ball.

[1]

(b) The ball is dropped from a position level with the top of the elastic thread as shown in Fig. 10.2.

A diagram has been removed due to third party copyright restrictions

Details: A diagram of a tennis ball attached to an elastic thread with the tennis ball in a position level with the top of the elastic thread

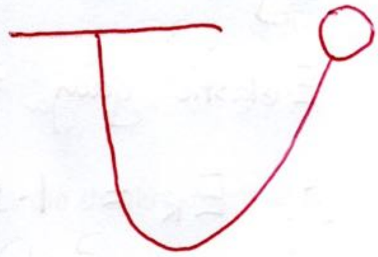


Fig. 10.2

Fig. 10.3 shows the graph of the resultant force on the ball as the distance fallen increases.

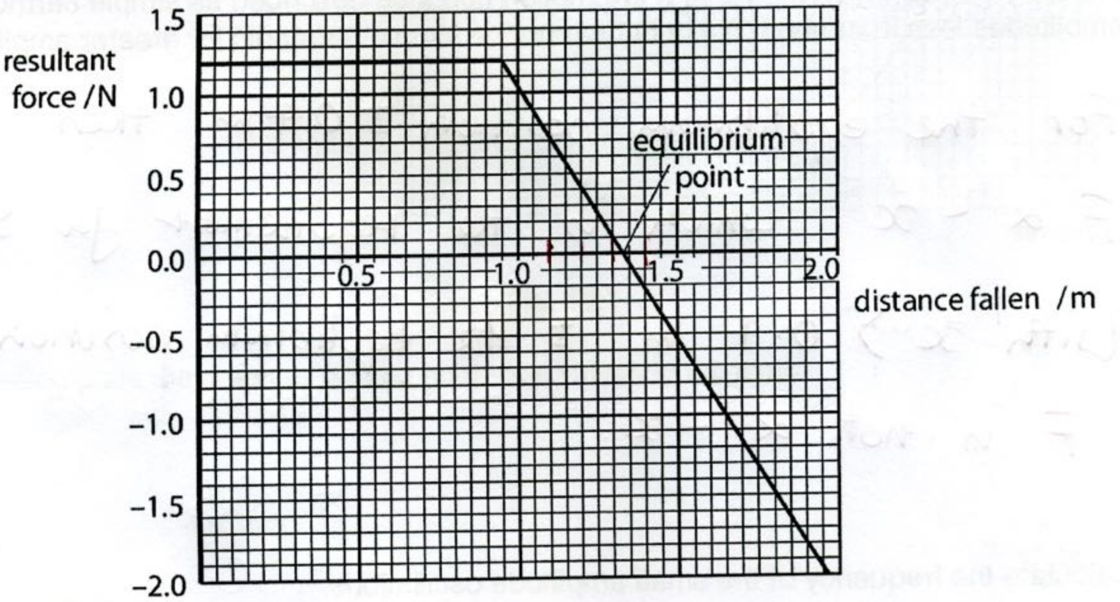


Fig. 10.3

Using the graph or by calculation show that

(i) the kinetic energy of the ball when it has fallen 0.95 m is about 1 J.

$E_K = E_{grav}$ as elastic is not stretched

$= mg\Delta h = 0.12 \times 9.8 \times 0.95 = 1.12$ J

[1]

Question 10 is continued over the page.

(ii) the kinetic energy of the ball when it has fallen to the point of zero resultant force (equilibrium point) is about 1.3 J.

E_{grav} lost $= mg\Delta h$ $\quad 0.12 \times 9.8 \times 1.325 = 1.558$ J

$E_{elastic}$ gain $= \frac{1}{2}kx^2 = \frac{1}{2} \times 3.1 \times 0.375^2 = 0.218$ J

$\therefore E_K = 1.558 - 0.218 = 1.34$ J

[2]

(c) The ball oscillates vertically. The amplitude decreases and after a number of oscillations the ball comes to rest at the equilibrium point.

(i) Explain how the graph shows that the motion could be described as simple harmonic for amplitudes less than about 0.4 m but cannot be simple harmonic for greater amplitudes.

For the equlibrium position ± 0.4 m then $F \propto -x$ which is the requirement for SHM.

With $x > 0.4$ m ± the equlibrium position F is not $\propto -x$.

[4]

(ii) Calculate the frequency of the small amplitude oscillations.

$f = \frac{1}{T}$ & $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.12}{3.1}}$

$= 1.236$ s

$\therefore f = 1/1.236 = 0.81$ Hz

frequency = ... Hz [2]

[Total: 13]

10 This question is about the motion of a piston in a car engine.

Here are some data about the motion of the piston.

frequency f = 50 Hz
amplitude A = 0.050 m

The motion of the piston is represented in Fig. 10.1 where x is the displacement at time t of the piston from the mean position.

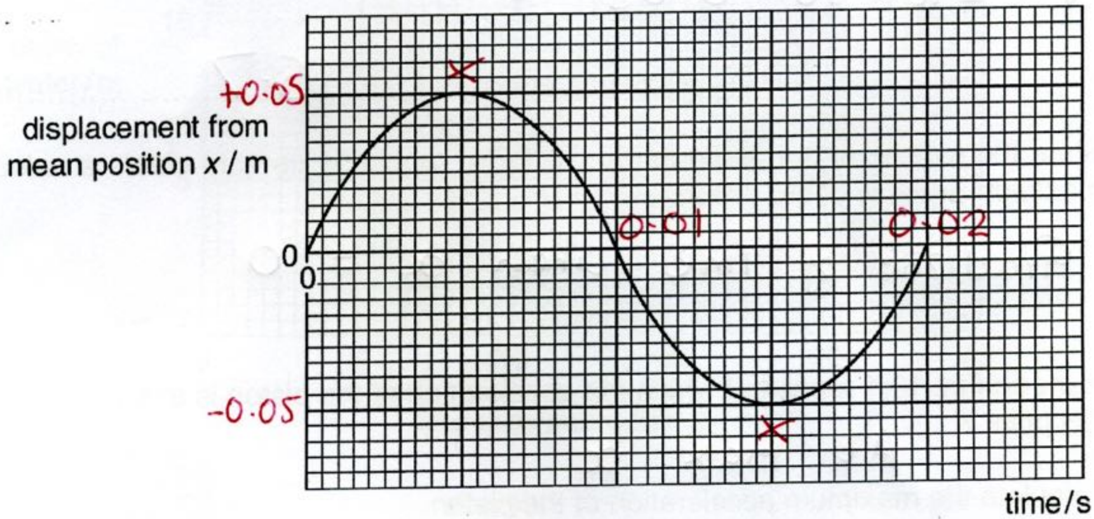


Fig. 10.1

(a) Complete the graph by adding numerical values to the axes.

[2]

(b) The equation of motion of the piston represented in Fig. 10.1 is

$x = A \sin(2\pi f t)$

where t is the time elapsed since the beginning of the motion.

Calculate the displacement of the piston at t = 0.013 s.

$x = 0.05 \sin(2\pi \times 50 \times 0.013) =$

displacement = − 0.040 m [2]

(c) The velocity of the piston v is given by the equation $v = 2\pi f A(\cos 2\pi ft)$.

(i) Explain how the equation shows that the maximum velocity of the piston is given by

maximum velocity = $2\pi f A$.

Max value for $\cos\theta = 1$

[1]

(ii) Calculate the maximum velocity of the piston.

$V = 2\pi \times 50 \times 0.05 =$

maximum velocity = 15.7 m s^{-1} [1]

(iii) State the feature of the graph in Fig. 10.1 that could be used to obtain a value for the maximum velocity of the piston.

Gradient of line when $x = 0$

[1]

(d) **(i)** Mark on Fig. 10.1 a point at which the acceleration of the piston is at a maximum. Label this point **X**. [1]

At max x

(ii) Calculate the maximum acceleration of the piston.

$a = -\omega^2 x$

$a_{max} = -\omega^2 A$

$\omega = 2\pi f$

$= 2^2 \pi^2 f^2 A$

$= 4 \times \pi^2 \times 50^2 \times 0.05 =$

acceleration = 4935 m s^{-2} [2]

4930 to 2SF

[Total: 10]