The Gravitational Field

Past Paper Question Booklet from G494

Field and potential	
for all fields	field strength $= -\dfrac{dV}{dr} \approx -\dfrac{\Delta V}{\Delta r}$
gravitational fields	$g = \dfrac{F}{m}$, $E_{grav} = -\dfrac{GmM}{r}$
	$V_{grav} = -\dfrac{GM}{r}$, $F = -\dfrac{GmM}{r^2}$

Jan 2010

1 Here is a list of units.

JK^{-1} $\qquad$ Jkg^{-1} $\qquad$ Nkg^{-1} $\qquad$ Ns

(a) Which one is a correct unit for gravitational potential?

$J\,kg^{-1}$ .... **[1]**

5 The planet Uranus has mass 8.7×10^{25} kg.

(a) Calculate the gravitational potential energy of a satellite of mass 43 kg in a circular orbit of radius 5.1×10^{7} m around Uranus.
$G = 6.7 \times 10^{-11}\,N\,kg^{-2}\,m^2$

$$E_{grav} = \frac{-GMm}{r} = \frac{-6.7\times10^{-11}\times 8.7\times10^{25}\times 43}{5.1\times10^{7}}$$

gravitational potential energy = -4.9×10^{9} .... J **[2]**

(b) Here are four statements **A**, **B**, **C** and **D** about the satellite in its circular orbit.

Which **one** of the statements is correct?

A Its orbital path is on an equipotential surface. ✓

B Its total energy must be greater than zero for a stable orbit. ×

C The gravitational potential of the satellite is its centripetal force. ×

D The velocity of the satellite is in the direction of the planet's gravitational field. ×

correct statement ....A........ **[1]**

10 This question is about using the motion of satellites to determine the mass of a planet.

Fig. 10.1 shows the path followed by the Moon as it orbits the Earth.

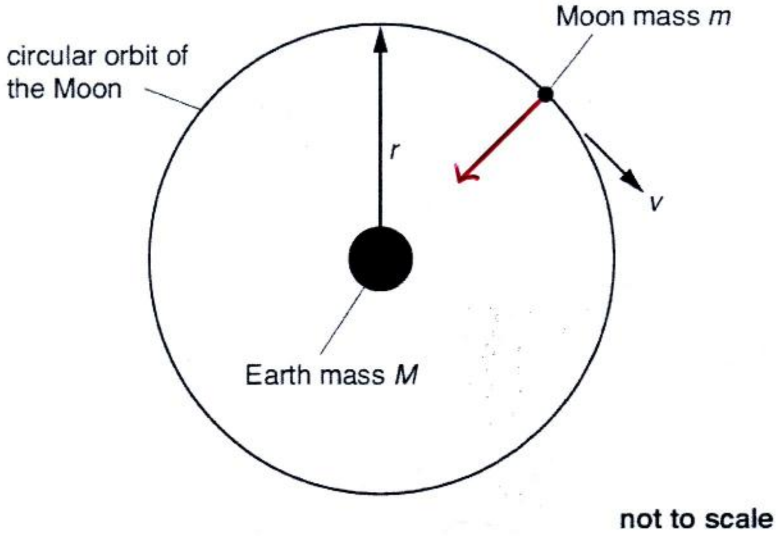


Fig. 10.1

(a) The average distance r from the centre of the Earth to the centre of the Moon is 3.8×10^8 m.

(ii) The Moon takes 27 days to make one complete orbit of the Earth. Show that the speed v of the Moon is about $1000\,\text{m}\,\text{s}^{-1}$.

$$v = \frac{\Delta s}{\Delta t} = \frac{2\pi \times 3.8 \times 10^8}{27 \times 24 \times 3600} = 1023\ \text{ms}^{-1}$$

[2]

(b) The Moon follows a circular orbit because it is accelerated by the gravitational field g of the Earth.

(i) Draw an arrow on Fig. 10.1 to show the direction of this acceleration. **[1]**

(ii) Explain why this acceleration does not change the speed of the Moon in its orbit around the Earth.

The acceleration is at right angles to the orbital motion. / The force is normal to the displacement so no work is done. **[1]**

(iii) The gravitational force of the Earth upon the Moon is the centripetal force acting on the Moon. Use this idea to calculate the mass of the Earth M from the values of v and r given above.

$G = 6.7 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.

$$F = \frac{mv^2}{r} = \frac{GMm}{r} \quad \therefore \quad v^2 = \frac{GM}{r}$$

$$M = \frac{v^2 r}{G} = \frac{1023^2 \times 3.8 \times 10^8}{6.7 \times 10^{-11}}$$

$M =$ ......5.9×10^{24}...... kg **[2]**

June 2010

1 Here is a list of units.

J kg^{-1} K^{-1} **J kg^{-1}** **J K^{-1}** **N kg^{-1}** **N s**

(a) Which one is a correct unit for gravitational field strength?

......N kg^{-1}...... **[1]**

13 This question is about the motion of Halley's comet.

Fig. 13.1 represents the highly elliptical path of Halley's comet in its 76 year orbit of the Sun.

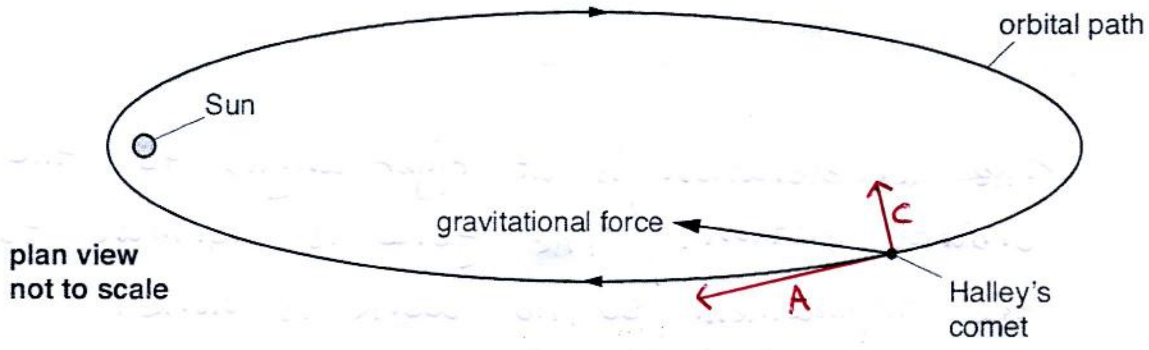


Fig. 13.1

(a) Fig. 13.1 shows the direction of the gravitational force on the comet.

(i) Draw an arrow on Fig. 13.1 to show the component of the gravitational force on the comet which changes its **speed**. Label it **A**. **[1]**

(ii) Draw an arrow on Fig. 13.1 to show the component of the gravitational force on the comet which changes its **direction**. Label it **C**. **[1]**

(b) At its closest approach to the Sun, the comet is moving at a speed of 54.6 km s^{-1}.

(i) Show that the **kinetic** energy per unit mass is about 1.5 GJ kg^{-1}.

$E_K = \frac{1}{2}mv^2$

$\frac{E_K}{m} = \frac{1}{2}v^2 = \frac{1}{2} \times (54.6 \times 10^3)^2$ **[2]**

$= 1.49 \times 10^9 \text{ J Kg}^{-1}$

$= 1.49 \text{ GJ Kg}^{-1}$

(ii) The distance from the Sun to the comet is 8.82×10^{10} m at its closest approach. Show that the **total** energy per unit mass is about $-20\,\mathrm{MJ\,kg^{-1}}$.

$G = 6.67 \times 10^{-11}\ \mathrm{N\,m2\,kg^{-2}}$
$M_s = 2.00 \times 10^{30}\ \mathrm{kg}$

$$V_{grav} = \frac{-GM}{r} = \frac{-6.67\times10^{-11}\times2.00\times10^{30}}{8.82\times10^{10}} = -1.51\times10^{9}\ \mathrm{J\,kg^{-1}}$$

$$V_{TOTAL} = V_{KE} + V_{GRAV} = 1.49\times10^{9} + -1.51\times10^{9}$$
$$= -2\times10^{7}\ \mathrm{J\,kg^{-1}} = -20\ \mathrm{MJ\,kg^{-1}}$$

[2]

(iii) When it is furthest from the Sun, the comet is 5.3×10^{12} m away from the Sun.

Calculate the speed of the comet at this distance.

In your answer you should show your method clearly and completely.

Due to conservation of energy V_{TOTAL} is still $-2\times10^{7}\ \mathrm{J\,kg^{-1}}$

$$V_{GRAV} = \frac{-GM}{r} = \frac{-6.67\times10^{-11}\times2.00\times10^{30}}{5.3\times10^{12}} = -25.2\ \mathrm{J\,kg^{-1}}$$

$$V_{TOTAL} = V_{KE} + V_{GRAV} \therefore V_{KE} = V_{TOT} - V_{GRAV} = -2\times10^{7} - -25.2\times10^{6}$$

$$V_{KE} = 5.2\times10^{6}\ \mathrm{J\,kg^{-1}} = \frac{1}{2}v^2$$

$\therefore v = \sqrt{2V_{KE}} = \sqrt{2\times5.2\times10^{6}}$ = speed = 3.2×10^{3} m s^{-1} **[3]**

Jan 2011

2 The graph of Fig. 2.1 shows how the gravitational field strength g of a planet varies with the distance r from its centre.

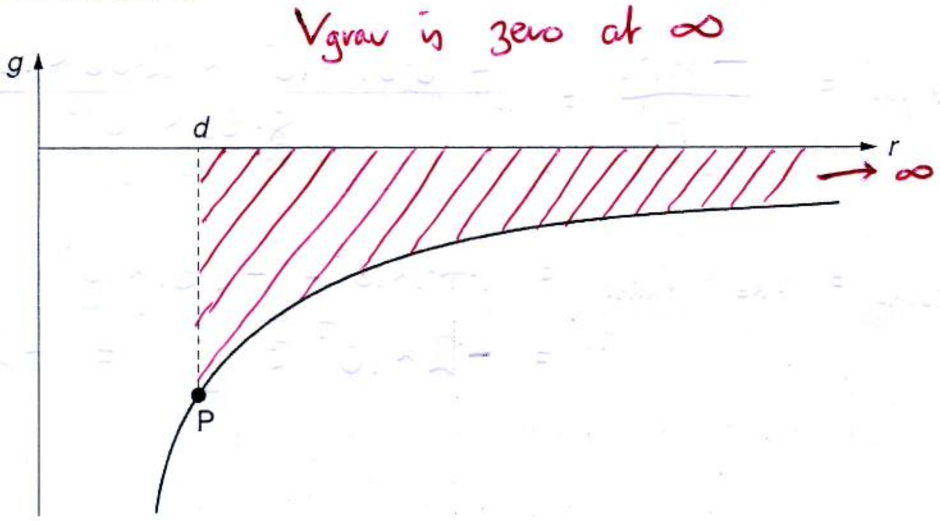


Fig. 2.1

Here are four suggested ways of using the graph to calculate the gravitational potential of a spacecraft at point P, when it is a distance d from the centre of the planet.

Put a tick (✓) in the box next to the correct calculation.

The gravitational potential is ...

... the gradient of the curve at P when $r = d$.	☐
... the reciprocal of the gradient of the curve at P when $r = d$.	☐
... the area between the curve and the r axis from $r = 0$ to $r = d$.	☐
... the area between the curve and the r axis from $r = d$ to $r = \infty$.	✓

[1]

11 This question is about placing a satellite in orbit around the planet Mars.

Fig. 11.1 shows the path followed by the satellite.

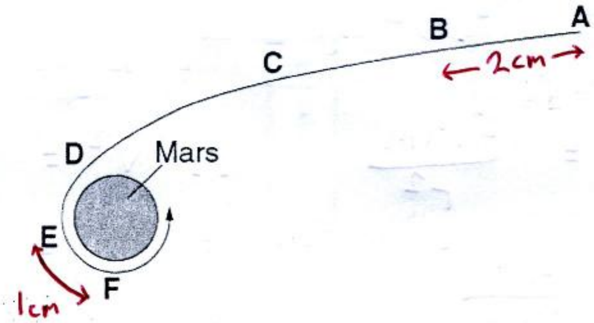


Fig. 11.1

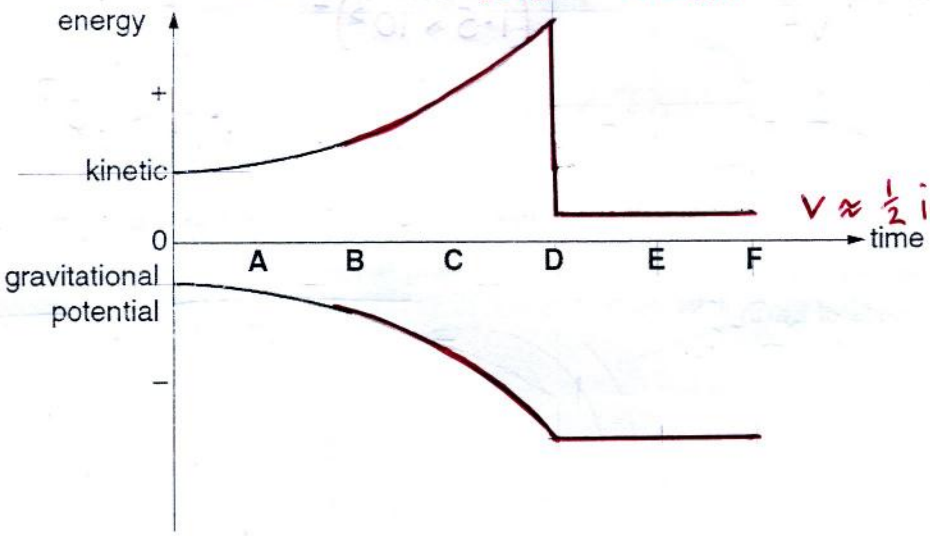
The satellite falls freely from **A** to **D** with its rocket thrusters turned off.

The rockets are fired briefly at point **D** to slow the satellite down and place it in a circular orbit.

(a) The points labelled **A** to **F** on the path are separated by the same interval of time.

Use Fig. 11.1 to complete the sketch graphs of Fig. 11.2 for the variation with time of the kinetic energy and gravitational potential energy of the satellite. The graphs have been started for you. **[4]**

$\Delta E_K = -\Delta E_{GRAV}$



$v \approx \frac{1}{2}$ initial

(c) **(i)** By equating the gravitational force on a satellite with the centripetal force required for a circular orbit of radius r around a planet of mass M, show that r is given by

$$r = \frac{GM}{v^2}$$

where G is the gravitational force constant and v is the speed of the satellite.

$$F = \frac{mv^2}{r} = \frac{GMm}{r^2} \quad \therefore \; v^2 = \frac{GM}{r}$$

$$\therefore \; r = \frac{GM}{v^2}$$

[2]

(ii) Calculate the radius r of the satellite's orbit around Mars for an orbit speed of $1.5 \times 10^3\,\mathrm{m\,s^{-1}}$.

$G = 6.7 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$

$M = 6.5 \times 10^{23}\,\mathrm{kg}$

$$r = \frac{GM}{v^2} = \frac{6.7\times10^{-11} \times 6.5\times10^{23}}{(1.5\times10^3)^2} =$$

$r =$ ....1.9×10^7.... m **[1]**

June 2011

5 Fig. 5.1 shows some equipotential surfaces drawn **inside** the Earth at equal intervals of potential.

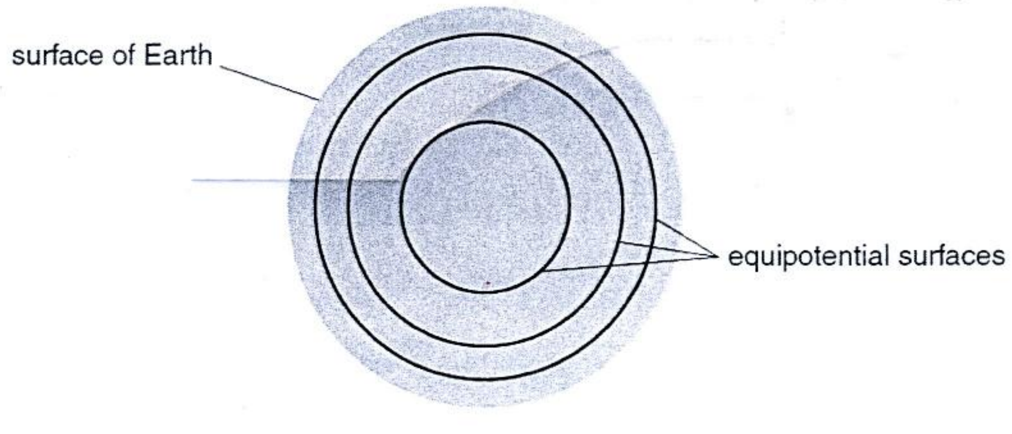


Fig. 5.1

State how the diagram shows that the gravitational field strength falls as you move from the surface towards the centre of the Earth.

The equipotential surfaces get further apart.

[1]

10 This question is about using the Moon as a source of raw materials for use on the Earth.
It has been proposed to launch metal extracted from ores on the Moon's surface towards the Earth by a cannon.

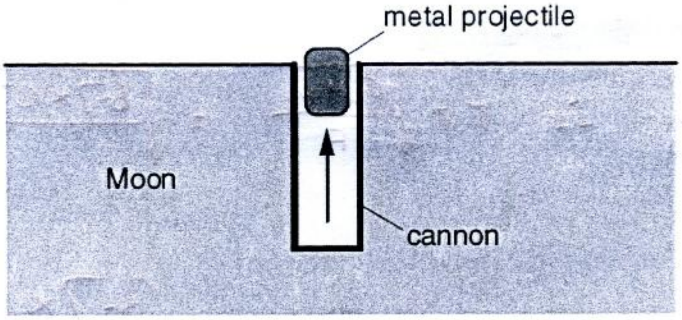


Fig. 10.1

(a) The metal projectile is subjected to a steady resultant force of 5.80×10^7 N for a time of 0.100 s by the cannon, starting from rest.
The mass of the projectile is 2.50×10^3 kg.

(i) By calculating the momentum of the projectile as it leaves the cannon, show that the projectile has a kinetic energy of 6.73×10^9 J as it leaves the cannon.

$$\Delta mv = F\Delta t = 5.8\times10^7 \times 0.100 = 5.8\times10^6 \text{ kgms}^{-1}$$

$$v = {\Delta mv}/{m} = 5.8\times10^6 / 2.5\times10^3 = 2320 \text{ ms}^{-1}$$

$$E_K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\times 2.5\times10^3 \times 2320^2 = 6.73\times10^9 \text{ J}$$

[3]

(ii) By calculating the gravitational potential energy of the projectile at the surface of the Moon show that the **total** energy of the projectile as it leaves the cannon is about -6×10^8 J.
Ignore the effect of the Earth.

mass of Moon = 7.4×10^{22} kg
radius of Moon = 1.7×10^6 m
$G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}

$$E_{grav} = \frac{-GMm}{r} = \frac{-6.7\times10^{-11}\times 7.4\times10^{22}\times 2.5\times10^3}{1.7\times10^6}$$

$$= -7.29\times10^9 \text{ J}$$

[2]

$$E_{TOTAL} = E_{KE} + E_{GRAV} = 6.73\times10^9 + -7.29\times10^9$$

$$= -5.6\times10^8 \text{ J}$$

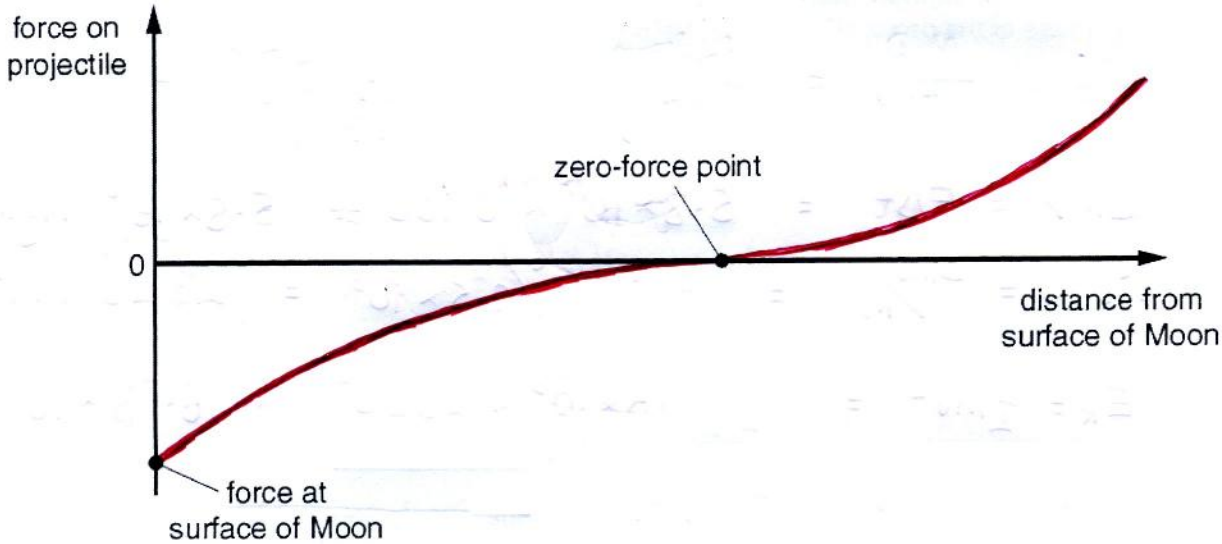
(b) The projectile is just able to reach the **zero-force point** between the Moon and the Earth. This is where the gravitational force of the Earth is equal and opposite to that of the Moon.

(i) At the zero-force point, the gravitational force of the Moon on the projectile is 9.0 N. Calculate the distance of the zero-force point from the centre of the Moon.

$$F = \frac{GMm}{r^2} \quad \therefore \quad r = \sqrt{\frac{GMm}{F}} = \sqrt{\frac{6.7\times10^{-11}\times 7.4\times10^{22}\times 2.5\times10^{3}}{9}} =$$

distance = 3.7×10^{7} m **[2]**

(ii) On the axes of Fig. 10.2, sketch how the gravitational force on the projectile varies as the projectile moves from the surface of the Moon towards the Earth.



[2]

Fig. 10.2

(c) The calculations in **(a)** which ignore the presence of the Earth suggest that the projectile does not have enough kinetic energy to reach the zero-force point. Explain how the presence of the Earth does allow the projectile to reach the zero-force point.

The gravitational attraction from the Earth extends out to the Moon (and beyond) and will reduce the resultant decelerating force on the projectile.

[2]

Jan 2012

4 Fig. 4.1 shows a satellite in a circular orbit around a planet.

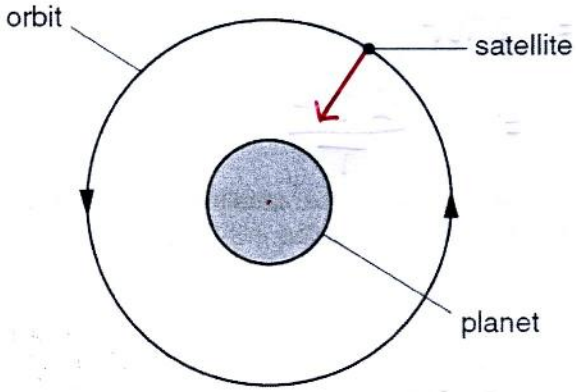


Fig. 4.1

(a) Draw an arrow on Fig. 4.1 to represent the resultant force on the satellite. **[1]**

(b) State why this force does not do any work on the satellite.

The force is at right angles (normal to) the direction of motion (displacement) **[1]**

10 This question is about the energy of a communications satellite in orbit around the Earth.

(a) The satellite is launched into a circular orbit of radius r with an orbit time T. Write down an expression for the speed v of the satellite in terms of r and T.

$$\Delta s = C = 2\pi r$$

$$v = \Delta s / \Delta t = \frac{2\pi r}{T}$$

[1]

(b) By equating the centripetal force on the satellite of mass m with its gravitational attraction to the Earth of mass M, show that the radius r of its orbit is given by the expression

$$r^3 = \frac{GMT^2}{4\pi^2}.$$

$$F = \frac{mv^2}{r} = \frac{GMm}{r^2} \quad \therefore \quad v^2 = \frac{GM}{r}$$

$$v = \frac{2\pi r}{T} \quad \therefore \quad v^2 = \frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$\therefore \quad r^3 = GMT^2/4\pi^2$$

[2]

(c) A useful communications satellite has an orbit time of 24 hours (8.6×10^4 s). Show that the radius r of its orbit is about 4×10^7 m.

mass of Earth = 6.0×10^{24} kg
$G = 6.7 \times 10^{-11}\, m^3 kg^{-1} s^{-2}$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{6.7\times10^{-11} \times 6\times10^{24} \times (8.6\times10^4)^2}{4\pi^2}}$$

$$= 4.22 \times 10^7 \text{ m}$$

[1]

(d) Calculate the gravitational potential energy of a satellite of mass 4.7×10^2 kg when it is in this orbit.

$$E_{grav} = \frac{-GMm}{r} = \frac{-6.7\times10^{-11}\times 6\times10^{24}\times 4.7\times10^{2}}{4.2\times10^{7}}$$

gravitational potential energy = -4.5×10^{9} J **[3]**

(e) The speed of the satellite in its orbit is $3.1 \times 10^3\,\mathrm{m\,s^{-1}}$.
Calculate the total energy of the satellite.

$$E_{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\times4.7\times10^{2}\times(3.1\times10^{3})^2 = 2.26\times10^{9}\ \mathrm{J}$$

$$E_{TOTAL} = E_{KE} + E_{GRAV} = 2.26\times10^{9} + -4.5\times10^{9} =$$

total energy = -2.24×10^{9} J **[2]**

June 2012

1 Here is a list of units.

$J\,kg^{-1}$ $N\,kg^{-1}$ $J\,kg^{-1}\,K^{-1}$ $N\,s$

(a) Which is a correct unit for gravitational potential?

answer ....$J\,kg^{-1}$........ **[1]**

8 Fig. 8.1 shows a large massive star in cross-section.

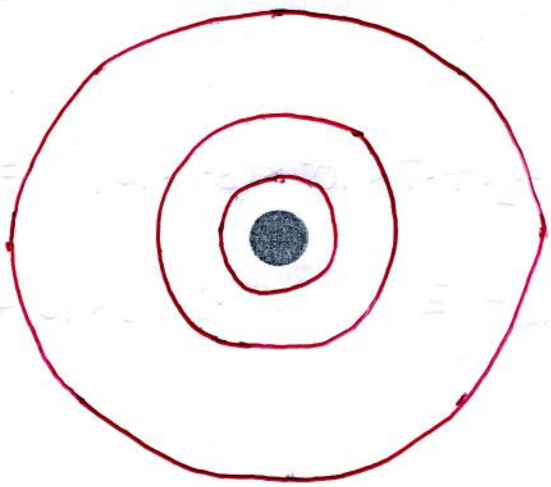


Fig. 8.1

Sketch **three** equipotentials around the star, each separated from the next by the same potential difference. **[1]**

13 This question is about the satellites in low altitude orbit around the Earth which provide the satellite telephone service.

(a) Each satellite follows a circular orbit centred on the Earth. By equating the centripetal force on a satellite of mass m to the gravitational force from the Earth of mass M, show that the speed of the satellite v when the orbit radius is r is given by

$$v^2 = \frac{GM}{r}.$$

$$F = \frac{mv^2}{r} = \frac{GMm}{r^2} \xrightarrow[r \,\&\, m]{\text{cancel}} v^2 = \frac{GM}{r}$$

[2]

(b) Show that the **total** energy $E = E_{kinetic} + E_{gravitational}$ of the satellite is given by

$$E = -\frac{GMm}{2r}.$$

$$E = \frac{1}{2}m\underbrace{v^2}_{=\frac{GM}{r}} + \frac{-GMm}{r} = \frac{mGM}{2r} - \frac{2GMm}{2r}$$

$$= \frac{-GMm}{2r}$$

[2]

(c) On the axes of Fig. 13.1, sketch a graph to show how E depends on r for a satellite in orbit, when r is greater than R, the radius of the Earth. [1]

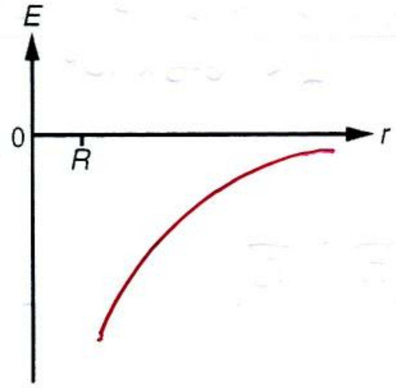
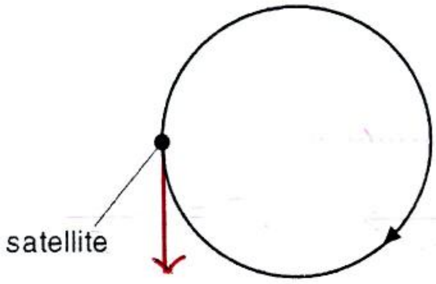


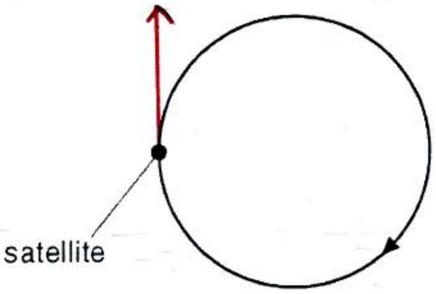
Fig. 13.1

(d) At the end of its lifetime, a rocket engine on the satellite ignites to slow it down and drop it to a lower orbit. Figs. 13.2 and 13.3 show the satellite in its orbit. The arrows show the direction of the satellite's orbit just before the engine ignites.



draw the direction of the force

Fig. 13.2



draw the direction of the gas

Fig. 13.3

(i) Draw an arrow on Fig. 13.2 to show the direction of the force on the spacecraft from the rocket engine when it ignites. **[1]**

(ii) Draw an arrow on Fig. 13.3 to show the direction of the gas from the rocket engine when it ignites. Equal and opposite **[1]**

(iii) Calculate the work done by the rocket engines to reduce the orbit radius from 7.2×10^6 m (1) to 6.5×10^6 m. (2)

$G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}
$M = 6.0 \times 10^{24}$ kg
$m = 6.9 \times 10^2$ kg

$$E = \frac{-GMm}{2r}$$

$$E \text{ at } (1) = \frac{-6.7 \times 10^{-11} \times 6 \times 10^{24} \times 6.9 \times 10^2}{2 \times 7.2 \times 10^6} = -1.93 \times 10^{10} \text{ J}$$

$$E \text{ at } (2) = \frac{-6.7 \times 10^{-11} \times 6 \times 10^{24} \times 6.9 \times 10^2}{2 \times 6.5 \times 10^6} = -2.13 \times 10^{10} \text{ J}$$

$$\Delta E = (-2.13 - -1.93) \times 10^{10}$$

Or use $\Delta E = \frac{-GMm}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

work done = -2.0×10^9 J **[2]**

Ans $= -2.1 \times 10^9$ J with no rounding

Jan 2013

6 The graph of Fig. 6.1 shows how the gravitational field strength g of a uniform sphere of radius R varies with the distance r from the centre of the sphere.

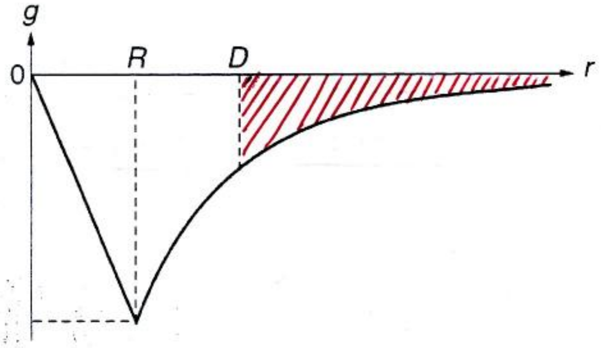


Fig. 6.1

(a) Shade the area of the graph which could be used to estimate the gravitational potential at a distance D from the centre of the sphere. **[1]**

(b) Here are some deductions from the graph about the gravitational field.
Put a tick (✓) next to the **one** correct deduction.

The field strength can never be zero. it is at $r = 0$ & ∞ ☐

The field changes direction at the surface of the sphere. ☐

The field direction is always towards the centre of the sphere. ✓ **[1]**

10 This question is about the gravitational field around the Earth, shown in Fig. 10.1.

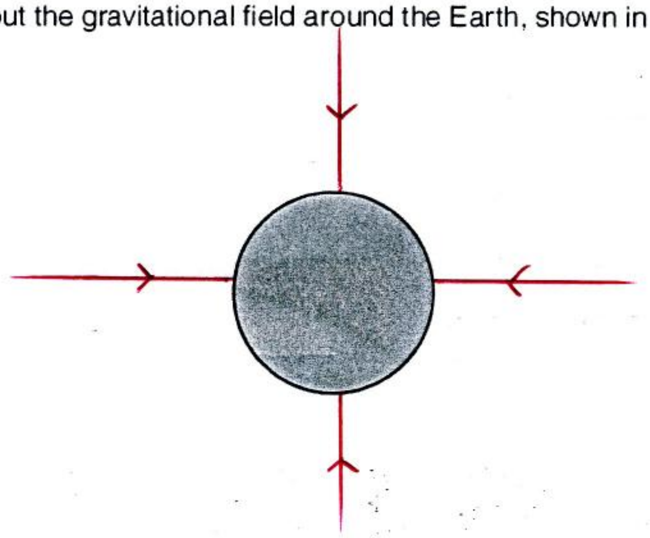


Fig. 10.1

(a) Draw four arrowed lines on Fig. 10.1 to represent the gravitational field above the Earth's surface. **[2]**

(b) The gravitational field strength at the Earth's surface, 6.4×10^6 m from its centre, is $9.8\,\text{N kg}^{-1}$. Show that the mass of the Earth is about 6×10^{24} kg.

$G = 6.7 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$

$$g = GM/r^2$$

$$\therefore M = gr^2/G = \frac{9.8 \times (6.4 \times 10^6)^2}{6.7 \times 10^{-11}} = 5.99 \times 10^{24}\ \text{kg}$$

[2]

(c) The Moon orbits the Earth, moving in a circle of radius 3.8×10^8 m.

(i) Explain why the value of the centripetal acceleration of the Moon in its orbit is equal to the gravitational field strength of the Earth at that distance.

The centripetal force for the circular motion is provided by the Earth's gravitational Force.

$$a = \frac{v^2}{r} = \frac{GM}{r^2} = g$$

[2]

(ii) Use the centripetal acceleration of the Moon to calculate the speed of the Moon in its orbit.

$$\frac{v^2}{r} = \frac{GM}{r^2} \quad \therefore \quad v = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{3.8 \times 10^{8}}} = 1026 \text{ ms}^{-1}$$

speed = 1.03×10^3 ms^{-1} **[3]**

June 2013

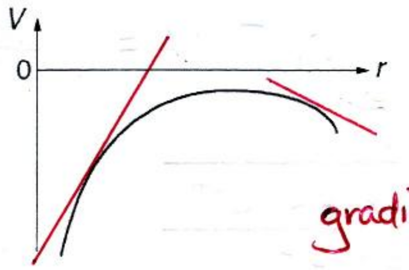
1 Here is a list of units.

J kg^{-1} **N m** **N kg^{-1}** **J m^{-1}**

(a) Which **one** is a correct unit for gravitational potential?

answer J kg^{-1} **[1]**

7 The graph of Fig. 7.1 shows how the gravitational potential V changes as you move from the surface of a planet to the surface of its moon, where r is the distance from the centre of the planet.



(gradient changes from +ve to −ve force must change direction.)

gradient = $g \propto F$

Fig. 7.1

Which of the graphs of Fig. 7.2 shows how the gravitational force F on you changes as you make the same journey?

$F \propto \frac{1}{r^2}$ so can't be straight line.

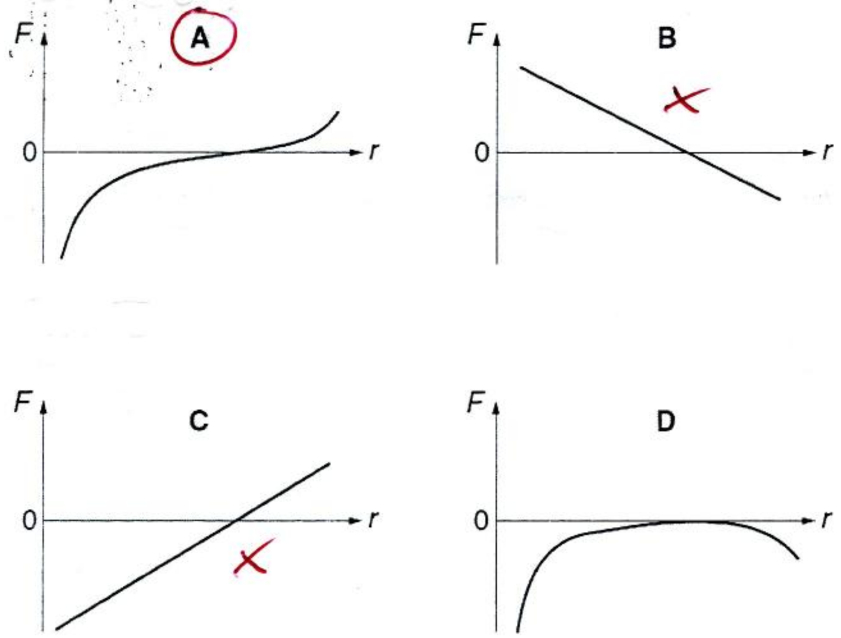


Fig. 7.2

answer ....A.......... **[1]**

12 Fig. 12.1 shows the circular orbit of a satellite **S** around a planet **P**.

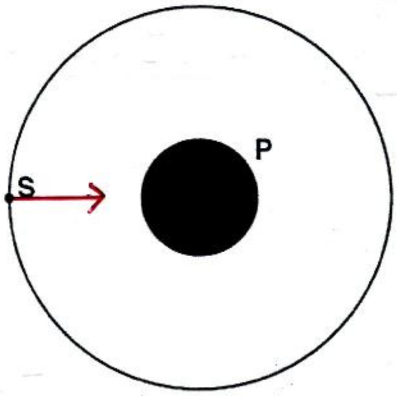


Fig. 12.1

(a) **(i)** Draw an arrow on Fig. 12.1 to show the direction of the planet's gravitational field at **S**. **[1]**

(ii) Explain why the force due to this gravitational field does not change the speed of the satellite.

The force is at right angles to the orbital motion.

[2]

(iii) By considering the centripetal force on the satellite, calculate its speed *v*.

mass of planet = 4.8×10^{23} kg

radius of orbit = 6.1×10^{6} m

$G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v = \sqrt{GM/r} = \sqrt{\frac{6.7\times10^{-11} \times 4.8\times10^{23}}{6.1\times10^{6}}} = 2.3\times10^{3}\ \text{ms}^{-1}$$

v = 2.3×10^{3} m s^{-1} **[3]**

(iv) Calculate the gravitational potential energy of the satellite **S**.

mass of satellite = 5.7×10^3 kg

$$E_{grav} = \frac{-GMm}{r} = \frac{-6.7\times10^{-11}\times 4.8\times10^{23}\times 5.7\times10^{3}}{6.1\times10^{6}}$$

gravitational potential energy = -3.0×10^{10} J **[2]**

(b) Two satellites, $\mathbf{S_1}$ and $\mathbf{S_2}$, of the same mass are also in circular orbits around **P**.

The radius of orbit for $\mathbf{S_1}$ is twice the radius of orbit for $\mathbf{S_2}$. Compare the kinetic energy of the two satellites.

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \quad \therefore \quad \frac{mv^2}{2} = \frac{GMm}{2r} = KE$$

$\therefore$ if radius is doubled the KE must halve.

$$2\,KE_{S1} = KE_{S_2}$$

[3]

June 2014

9 The asteroid belt between Mars and Jupiter contains a large number of rocks in circular orbit around the Sun.

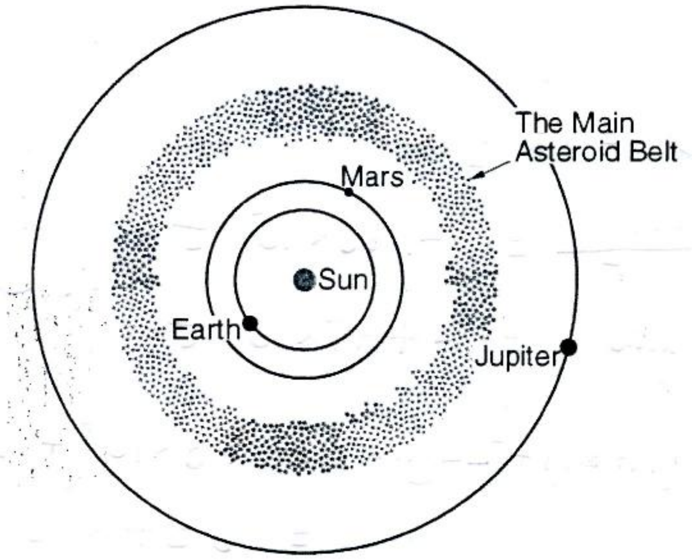


Fig. 9.1

(a) Show that the speed v of an asteroid of mass m in a circular orbit of radius r around the Sun of mass M is given by

$$v = \sqrt{\frac{GM}{r}}.$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \xrightarrow[m \,\&\, r]{\text{cancel}} v^2 = \frac{GM}{r}$$

$$\therefore v = \sqrt{\frac{GM}{r}}$$

[2]

(b) One particular asteroid in the asteroid belt is in a circular orbit of radius 3.6×10^{11} m.

(i) Show that it has a kinetic energy of about 10^{11} J.

mass of asteroid = 500 kg
mass of Sun = 2.0×10^{30} kg
$G = 6.7 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$

$$v^2 = \frac{GM}{r} \quad \therefore \quad \frac{1}{2}mv^2 = \frac{GMm}{2r} = KE$$

$$\therefore KE = \frac{6.7 \times 10^{-11} \times 2 \times 10^{30} \times 500}{2 \times 3.6 \times 10^{11}} = 9.3 \times 10^{10}\ \text{J}$$

[2]

(ii) It is believed that collisions between asteroids can put them into elliptical orbits which cross the Earth's orbit. These asteroids may then collide with the Earth, causing widespread damage.

If a collision changes the direction of motion of the asteroid in **(i)** **without** changing its kinetic energy, calculate its speed v when it crosses the Earth's orbit.

radius of the Earth's orbit = 1.5×10^{11} m

E_{grav} lost = KE gained

E_{grav} (initial) $= \frac{-GMm}{r} = -1.86 \times 10^{11}$ J

E_{grav} (final) = ditto $= -4.47 \times 10^{11}$ J

$\therefore \Delta E_{grav} = -2.61 \times 10^{11}$ J

E_K (final) = E_K (initial) + $^{-}\Delta E_{grav}$ $= 9.3 \times 10^{10} + 2.61 \times 10^{11}$

$= 3.54 \times 10^{11}$ J

$v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2 \times 3.54 \times 10^{11}}{500}}$

v = ...3.76×10^4...... m s^{-1} **[4]**

June 2015

1 Here is a list of units.

N m kg^{-1} **N kg^{-1}** **J m^{-1}** **kg m s^{-1}** **m s^{-1}**

(a) Which one is a correct unit for gravitational field strength?

......N kg^{-1}...... **[1]**

(b) Which one is a correct unit for gravitational potential?

......N m kg^{-1}...... **[1]**

Nm = J

(Fs = W)

9 Fig. 9.1 shows a binary star system. Both stars have the same mass and radius.

Fig. 9.1

Fig. 9.2 shows four different attempts to sketch a gravitational equipotential curve in the binary star system.
Which attempt **A**, **B**, **C** or **D** is correct?

At right angles to field lines

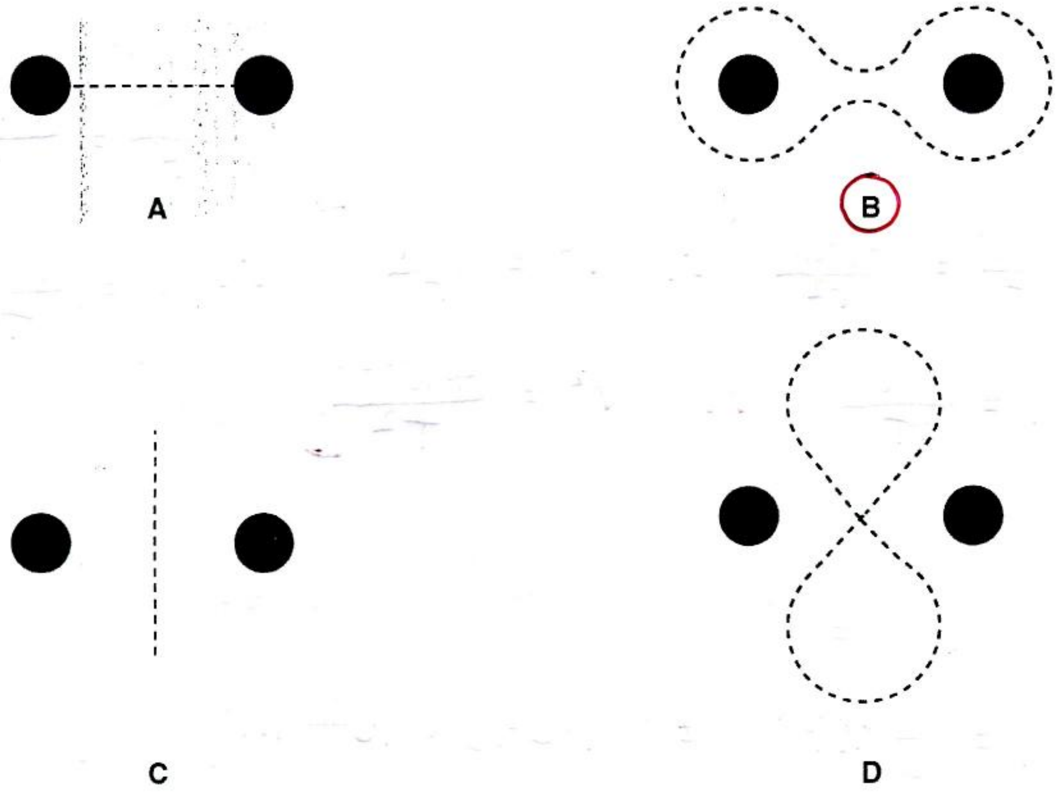


Fig. 9.2

answer ....B.......................... **[1]**

10 This question is about possible flaws in an attempt to estimate a value for the gravitational constant G using Earth-based observations.

The table contains the results of data required.

radius of Earth at equator	6.4×10^{6} m
mean density of surface rocks on Earth	2.7×10^{3} kg m^{-3}
period of Moon's orbit around the Earth	2.4×10^{6} s
time for a laser pulse fired at the Moon to return	2.5 s

(a) The Moon of mass m orbits the Earth of mass M in a circular path of radius r with a period T. By setting the centripetal force on the Moon equal to its gravitational attraction to the Earth, show that

$$G = \left(\frac{4\pi^2}{M}\right)\frac{r^3}{T^2}$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \quad \therefore \quad v^2 = \frac{GM}{r}$$

$$v = \frac{2\pi r}{T} \quad \therefore \quad v^2 = \frac{4\pi^2 r^2}{T^2}$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$\therefore \quad G = \frac{4\pi^2 r^3}{T^2 M}$$

[3]

(b) The mass M of the Earth can be estimated from its radius R and its density ρ.

Use the data for radius and density in the table to show that they give a value of M which is about 3×10^{24} kg.

$$V = \frac{4}{3}\pi r^3 = 1.098 \times 10^{21}\ m^3$$

$$m = V\rho = 1.098 \times 10^{21} \times 2.7 \times 10^{3}$$

$$= 2.96 \times 10^{24}\ kg$$

[2]

(c) The distance r from the centre of the Earth to the centre of the Moon can be estimated by timing the return of laser pulses reflected from the Moon's surface.
The relationship $r = \frac{c\Delta t}{2}$ can be used to estimate a value for r from the time interval Δt between the emission and return of a laser pulse.

(i) State **two** assumptions required for the relationship to be valid.

This question is from chapter 13.1

The speed of light is constant
The out & return journey times are equal
(can ignore the relatively small radii of E & M) **[2]**

(ii) Use the data from the table to calculate a value for r and hence show that the value of G obtained from **(a)** and **(b)** is very different from the accepted value of $6.7 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.

$c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$

$$r = \frac{3 \times 10^8 \times 2.5}{2} = 3.75 \times 10^8\,\mathrm{m}$$

$$G = \frac{4\pi^2 \times (3.75 \times 10^8)^3}{3 \times 10^{24} \times (2.4 \times 10^6)^2} =$$

$G =$ ....1.2×10^{-10}.... $\mathrm{N\,m^2\,kg^{-2}}$ **[2]**

(iii) Suggest a reason why your calculated value of G is very different from the accepted value. Justify your answer.

Value for G is too large as the value for M will be too small as the density of the mantle and core are greater than the crust. **[2]**

June 2016

10 This question is about the planet Jupiter and its satellite Io.

(a) The mean radius r of the orbit of Io is 4.2×10^8 m and its orbital period T is 43 hours.

(i) Show that Io's mean orbital speed v is about $20\,\text{km}\,\text{s}^{-1}$.

$$V = \frac{2\pi r}{T} = \frac{2\pi \times 4.2 \times 10^8}{43 \times 3600} = 1.70 \times 10^4\ \text{m}\,\text{s}^{-1}$$

[2]

(ii) By considering the centripetal force on Io from Jupiter, show that the mass of Jupiter M is given by

$$M = \frac{rv^2}{G}.$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \quad \therefore \quad v^2 = \frac{GM}{r}$$

$$\therefore \quad M = \frac{v^2 r}{G}$$

[2]

(iii) Calculate the mass of Jupiter.

$G = 6.7 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$

$$M = \frac{v^2 r}{G} = \frac{(1.7 \times 10^4)^2 \times 4.2 \times 10^8}{6.7 \times 10^{-11}} =$$

mass = 1.8×10^{27} kg **[1]**

(b) Io's orbit is actually an ellipse, shown exaggerated in Fig. 10.1.

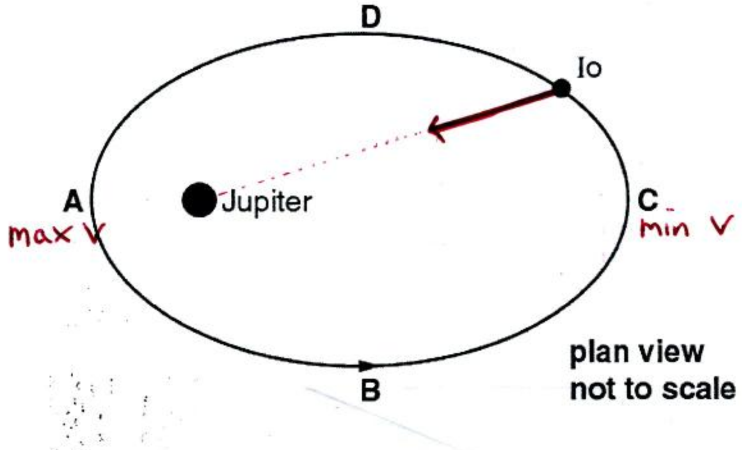


Fig. 10.1

(i) Draw an arrow on Fig. 10.1 to show the direction of the gravitational force on Io from Jupiter. **[1]**

(ii) On Fig. 10.2, sketch a graph to show how the speed of Io changes in one orbit around Jupiter.

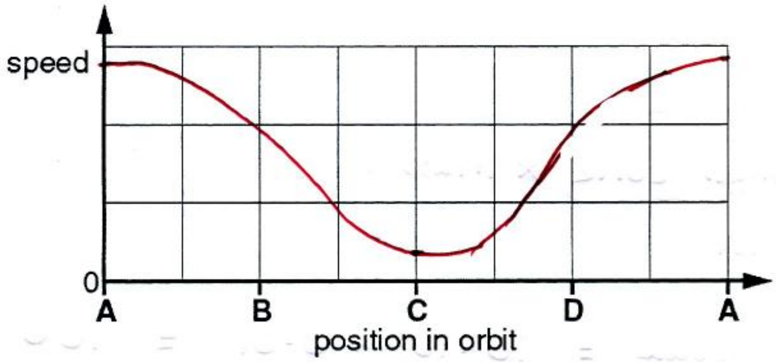


Fig. 10.2

[1]

(iii) Explain why the speed changes in this way.

$E_{TOTAL} = E_{KINETIC} + E_{GRAV} =$ constant (Energy is conserved)

As r gets smaller E_{grav} gets more negative (smaller) so E_K increases resulting in an increase in speed. **[2]**

June 2017

3 Fig. 3.1 shows the variation in gravitational field strength with distance from Ceres, the largest known asteroid.

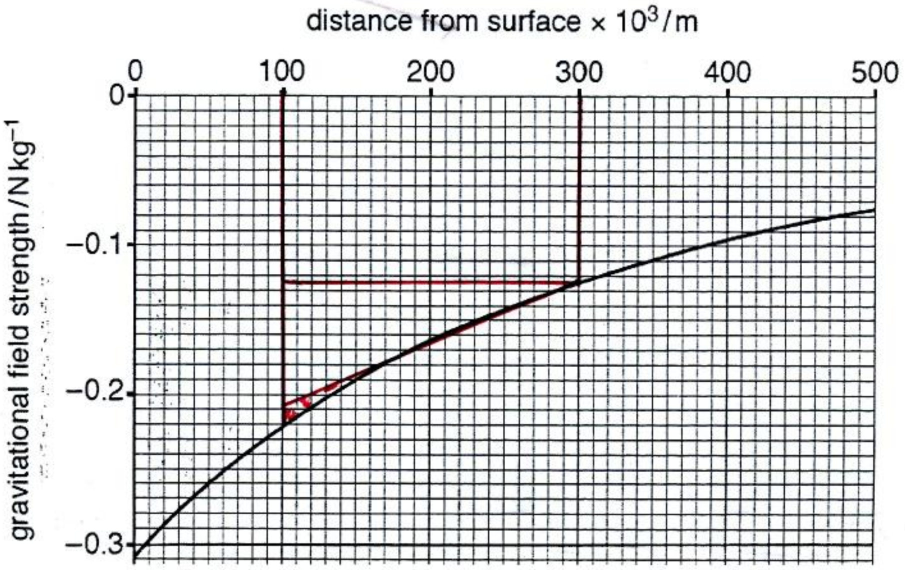


Fig. 3.1

Use the graph to estimate the energy required to move a mass of 2.0 kg from 100×10^3 m above the surface to 300×10^3 m above the surface.

$\Delta E = \underbrace{\text{area under curve}}_{\Delta V} \times \text{mass}$

energy = ... J **[2]**

Each small square $= 10 \times 10^3 \times 0.01 = 100 \text{ J kg}^{-1}$

No of squares $= 20 \times 12.5 + \frac{1}{2} \times 20 \times 8.5 + 3$

$= 338$ squares $= 3.38 \times 10^4 \text{ J kg}^{-1}$

$\Delta E = m \Delta V = 2 \times 3.38 \times 10^4$

$= 6.76 \times 10^4$ J

9 This question is about the gravitational field around a small, spherical asteroid. It is assumed that the asteroid is of uniform density.

(a) Fig. 9.1 shows some equipotential lines around the asteroid. Draw the gravitational field line through point **X**. **[2]**

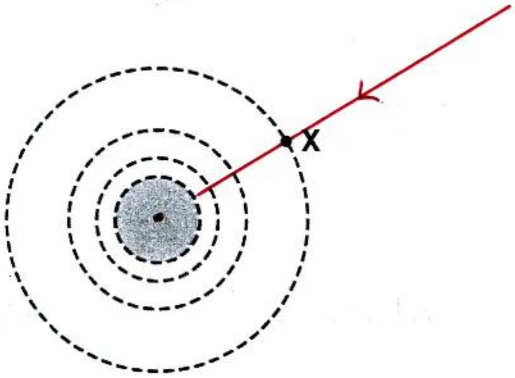


Fig. 9.1

(b) It is suggested that a space vehicle could land on the asteroid. Calculate the gravitational force on a vehicle of mass 2.9×10^2 kg on the surface of the asteroid.

Data: radius of asteroid = 1.45×10^5 m

mass of asteroid = 7.0×10^{19} kg

$$F = \frac{GMm}{r^2} = \frac{6.67\times10^{-11} \times 7\times10^{19} \times 2.9\times10^{2}}{(1.45\times10^{5})^{2}} =$$

force = 64.4 N **[2]**

(c) The asteroid is spinning, making one rotation every 320 minutes. The space vehicle is on the equator of the asteroid at a distance 1.45×10^5 m from the centre.

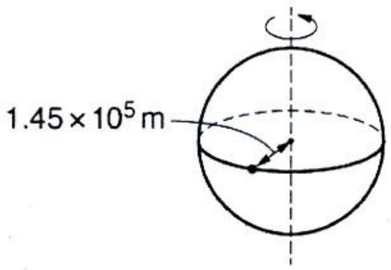


Fig. 9.2

(i) Calculate the force needed to keep the space vehicle on the surface of the spinning asteroid at the equator and explain why the space vehicle remains on the surface despite the rotation of the asteroid.

You should use technical terms correctly in your answer.

$F = \frac{mv^2}{r}$ $\quad V = \frac{2\pi r}{T} = \frac{2\pi \times 1.45\times10^5}{320\times60} = 47.5\ ms^{-1}$

$F = \frac{2.9\times10^2 \times 47.5^2}{1.45\times10^5} = 4.5\ N$

The vehicle remains on the surface as its weight of 64 N is more than enough to provide the centripetal force for its motion due to the spinning asteroid.

[4]

(ii) It is suggested that a less massive space vehicle would be less likely to remain on the surface of the asteroid. Comment on this suggestion.

There would be no difference.

$a = \frac{v^2}{r} = \frac{-GM}{r^2} = g$

a & g are independent of the mass of the vehicle

[2]

(A lighter vehicle requires less centripetal force)